Recent progress in flapping wing aerodynamics and aeroelasticity

W. Shyy a,*, H. Aono a, S.K. Chimakurthi a, P. Trizila a, C.-K. Kang a, C.E.S. Cesnik a, H. Liu b

a Department of Aerospace Engineering, University of Michigan, FKB 1320 Beal Avenue, Ann Arbor, MI 48109, USA
b Graduate School of Engineering, Chiba University, 1-33 Yayoi-cho, Chiba, Chiba 263-8522, Japan

A R T I C L E I N F O
Available online 13 February 2010

A B S T R A C T

Micro air vehicles (MAVs) have the potential to revolutionize our sensing and information gathering capabilities in areas such as environmental monitoring and homeland security. Flapping wings with suitable wing kinematics, wing shapes, and flexible structures can enhance lift as well as thrust by exploiting large-scale vortical flow structures under various conditions. However, the scaling invariance of both fluid dynamics and structural dynamics as the size changes is fundamentally difficult. The focus of this review is to assess the recent progress in flapping wing aerodynamics and aeroelasticity. It is realized that a variation of the Reynolds number (wing sizing, flapping frequency, etc.) leads to a change in the leading edge vortex (LEV) and spanwise flow structures, which impacts the aerodynamic force generation. While in classical stationary wing theory, the tip vortices (TiVs) are seen as wasted energy, in flapping flight, they can interact with the LEV to enhance lift without increasing the power requirements. Surrogate modeling techniques can assess the aerodynamic outcomes between two- and three-dimensional wing. The combined effect of the TiVs, the LEV, and jet can improve the aerodynamics of a flapping wing. Regarding aeroelasticity, chordwise flexibility in the forward flight can substantially adjust the projected area normal to the flight trajectory via shape deformation, hence redistributing thrust and lift. Spanwise flexibility in the forward flight creates shape deformation from the wing root to the wing tip resulting in varied phase shift and effective angle of attack distribution along the wing span. Numerous open issues in flapping wing aerodynamics are highlighted.

© 2010 Elsevier Ltd. All rights reserved.

Contents

1. Introduction ................................................................. 285
2. Equations and parameters of flapping wing dynamics ................................................. 286
2.1. Kinematics of flapping flight ........................................... 286
2.2. Governing equations .................................................. 287
2.3. Scaling laws .......................................................... 288
3. Key attribute of unsteady flapping wing aerodynamics ............................................. 290
3.1. Clap and fling ......................................................... 291
3.2. Rapid pitch rotation .................................................. 291
3.3. Wake capture ........................................................ 291
3.4. Delayed stall of leading edge vortex (LEV) ...................................................... 292
3.5. Tip vortex (TV) ..................................................... 293
3.6. Passive pitching mechanism .......................................... 293
4. Kinematics, wing geometry, Re, and rigid flapping wing aerodynamics ....................... 294
4.1. Single wing in forward flight condition ............................................. 294
4.2. Single wing in hovering flight condition .............................................. 296
4.3. Tandem wing in forward/hovering flight condition ......................................... 297
4.4. Implications of wing geometry ........................................... 297
4.5. Implications of wing kinematics ........................................... 298
4.6. Surrogate modeling for hovering wing aerodynamics ........................................ 298

Abbreviations: AoA, angle of attack; LEV, leading edge vortex; MAV, micro air vehicle; MTV, molecular tagging velocimetry
* Corresponding author. Tel: +1 734 936 0102.
E-mail address: weishyy@umich.edu (W. Shyy).

0376-0421/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.paerosci.2010.01.001
1. Introduction

Micro air vehicles (MAVs) have the potential to revolutionize our sensing and information gathering capabilities in areas such as environmental monitoring and homeland security. Numerous vehicle concepts, including fixed wing, rotary wing, and flapping wing, have been proposed [1–8]. As the size of a vehicle becomes smaller than a few centimeters, fixed wing designs encounter fundamental challenges in lift generation and flight control. There are merits and challenges associated with rotary and flapping wing designs. Fundamentally, due to the Reynolds number effect, the aerodynamic characteristics such as the lift, drag and thrust of a flight vehicle change considerably between MAVs and conventional manned air vehicles [1–8]. And, since MAVs are of light weight and fly at low speeds, they are sensitive to wind gust [1–9]. Furthermore, their wing structures are often flexible and tend to deform during flight. Consequently, the fluid and structural dynamics of these flyers are closely linked to each other. Because of the common characteristics shared by MAVs and biological flyers, the aerospace and biological science communities are now actively communicating and collaborating. Much can be shared between researchers with different training and background including biological insight, mathematical models, physical interpretation, experimental techniques, and design concepts.

In order to handle wind gust, object avoidance, or station keeping, highly deformed wing shapes and coordinated wing–tail movement in the biological flight are often observed. Understanding the aerodynamic, structural, and control implications of these modes is essential for the development of high performance and robust flapping wing MAVs for accomplishing desirable missions. Moreover, the large flexibility of the wings leads to complex fluid–structure interactions, while the kinematics of flapping and the spectacular maneuvers performed by natural flyers result in highly coupled nonlinearities in fluid dynamics, aeroelasticity, flight dynamics, and control systems.

Insect wing structures are inherently anisotropic due to their membrane–vein configurations, with the spanwise bending stiffness being approximately 1–2 orders of magnitude larger than the chordwise bending stiffness in a majority of insect species [10,11]. In general, the spanwise flexural stiffness scales with the third power of the wing chord, while the chordwise stiffness scales with the second power of the wing chord [10,11]. Insect wings exhibit substantial variations in aspect ratio and configuration but share a common feature of a reinforced leading edge. A dragonfly wing has more local variations in its structural composition and is more corrugated than the wing of a cicada or a wasp [1,12]. It has been shown in the literature [1–3,12] that wing corrugation increases both warping rigidity and flexibility. Furthermore, specific characteristic features have been observed in the wing structure of a dragonfly which help prevent fatigue fracture [1,12]. The thin nature of the insect wing skin structure makes it unsuitable for taking compressive loads, which may result in skin wrinkling and/or buckling, i.e., large local deformations that will interact with the flow. On the aerodynamics side, in a fixed wing set-up, wind tunnel measurements show that corrugated wings are aerodynamically insensitive to the Reynolds number variations, which is quite different from a typical low Reynolds number airfoil [1,4,6,7,12].

As highlighted above, biological flyers showcase desirable flight characteristics and performance objectives [1,12–23]. The strategies exhibited in nature have the potential to be utilized in the design of flapping wing MAVs [1–8,19–27]. In particular, wing flexibility is likely to have a significant influence on the resulting aerodynamics. Based on a literature survey, it was found that several questions in flexible wing aerodynamics have not been adequately addressed in existing literature, among which, the key ones include: (i) How do geometrically nonlinear effects and the anisotropy of the structure impact the aerodynamics characteristics of the flapping wing? (ii) How can flapping flight be stabilized passively via flexible structures?

Furthermore, even though the rigid wing aerodynamics have been explored in more detail than the flexible wing aerodynamics, several questions still remain, among which, the key ones include: (i) How can the unsteady flow features be manipulated to enhance performance? As the sizing, flapping kinematics, flapping frequency, and flight speed vary, which fluid physics mechanisms are important? (ii) How can the observations from high fidelity simulations or experimental studies be distilled into reduced order models so that they are fast enough to execute for MAV control development? Since all of the above are not necessarily independent topics, a comprehensive understanding of the role of flapping wing kinematics, aerodynamics, and flexibility is central to the success of future flapping wing MAV designs.

As evidenced in the references cited, a number of publications exist to address numerous aspects of these issues. Furthermore, recently, many researchers have taken serious efforts in investigating these topics. There seems to be a need to consolidate the fast developing information to help update and benefit the community. The purpose of this paper is to complement the recent work presented by Shyy et al. [1] to review the recent progress in flapping wing aerodynamics and aeroelasticity at low Reynolds numbers, namely, \((10^4)–(10^6)\). In addition to present established information, open issues in both aerodynamics/aeroelasticity are highlighted so as to encourage future community-wide efforts.

The rest of the paper is organized as follows:

Flapping wing kinematics, governing equations, and scaling laws are presented in Section 2. Unsteady flight mechanisms associated with flapping wings and frequently encountered in the literature are described in Section 3. A literature survey focusing on flapping wing aerodynamics and aeroelasticity is presented in Sections 4 and 5 while emphasizing computational efforts of the authors to highlight selected flapping wing physics. Finally, concluding remarks and areas warranting further study are made in Section 6.
2. Equations and parameters of flapping wing dynamics

Aerodynamics associated with flapping wings can be modeled using the unsteady Navier–Stokes (NS) equations. Nonlinear physics with multiple variables (velocity, pressure) and moving geometries are among the aspects of primary interest. Numerous flapping wing kinematics and scaling parameters related to the fluid dynamics and fluid–structure interactions exist. The flight regime of each flyer is characterized by these parameters.

2.1. Kinematics of flapping flight

The kinematics of flapping flight is composed of body and wing movements. As shown in Fig. 1, assuming that the wing and body are rigid, the body kinematics can be represented by the body angle ($\chi$) (inclination of the body), relative to the horizontal plane (e.g. the ground). The flapping wing kinematics can be described by three basic positional angles of the wing with the stroke plane ($\beta$): (i) flapping about the $x$-axis in the wing root-fixed coordinate system described by the positional or stroke angle ($\beta$), (ii) rotation of the wing about the $x$-axis in the wing root-fixed coordinate system described by the angle of attack ($\alpha$), (iii) rotation of the wing about the $y$-axis in the wing root-fixed coordinate system described by the angle of attack ($\gamma$). The stroke plane is defined by the wing base and the wing tip of the maximum and the minimum sweep positions. Examples of the time histories of three insects are shown in Fig. 2. The time histories of positional angle show approximately first-order sinusoidal curves. The time histories of elevation angle show high frequency component of the flapping frequency. On the other hand, the time histories of angle of attack include high frequency component of the flapping frequency and some of insects show asymmetric patterns per stroke. Moreover, the body angle and the stroke plane angle vary in accordance with the flight speed and flapping wing kinematics of biological flyers [1,28,29].

Due to the complexity of the aerodynamics associated with bio-mimicking kinematics (as shown in Figs. 1 and 2), building a description of the fundamental factors involved can benefit from simplified models. The simplified models referred to later (Section 4) in the paper remove the rotational (centripetal and Coriolis) aspects while retaining vortex dynamics important in flapping wing flight. While the combination of pitching and translation (versus flapping about a pivot point) (see Fig. 3) of the entire wing are not found in a biological flyer, the motions used provide a basis for more complex analysis and are feasible mechanical designs.

The motions of the wing can be described by sinusoidal translation (along the $X$-axis) and pitching (about the $Y$-axis) with a...)
the case of simple harmonic motion. It is given by the following expression [30,31]:

$$\alpha_e = \tan^{-1} \left( -\frac{h}{U_{\infty}} \right) - \alpha$$

(1)

where the effective angle of attack \( \alpha_e \) is measured at reference points of the airfoil, \( x \) is the angle of attack, \( U_{\infty} \) is freestream velocity or flight velocity, and \( h \) is the velocity due to plunge motion, respectively. It may be noted that in the unsteady motion of flexible wing structures, the velocities (\( h \)) not only include the rigid body plunge and pitch motions, respectively but also those due to bending and twist deformations. In such a case, the angle of attack (\( \alpha \)) can be defined simply as the angle between the line joining the leading and trailing edges of an airfoil section and the direction of freestream. If the structure behaves as a beam, the effective angle of attack will remain the same at each point along the chord (the angle of attack still varies along the chord due to chordwise variation of induced flow). It is clearly not the case if the structure behaves like a plate or a shell. Effective angles of attack vary through the chord due to plate-like deformations. The angle at three-quarter chord then may become a representative sectional effective angle of attack [30]. Fig. 4 illustrates the effective angle of attack due to prescribed plunge motion (reference point is three-quarter chord from the trailing edge [31]), and prescribed plunge motion (reference point is the leading edge) with chordwise deformation [32].

### 2.2. Governing equations

The governing equations of fluid are the unsteady, incompressible 3-D NS equations and the continuity equation, which are expressed in vector form as follows:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\frac{1}{\rho_f} \nabla p + \frac{\mu}{\rho_f} \nabla^2 \mathbf{U},$$

\[ \nabla \cdot \mathbf{U} = 0 \tag{2} \]

where \( \rho_f \) is the fluid density, \( \mu \) is the dynamic viscosity coefficient, \( \mathbf{U} = (U \ V \ W) \) is the velocity vector of the fluid, \( t \) is the time, \( \mathbf{X} = (X \ Y \ Z) \) is the position vector of the fluid based on the inertial frame, \( \nabla \) is the gradient operator with respect to \( \mathbf{X} \), and \( p \) is the pressure, respectively.

Assuming an isotropic plate-like flapping wing structure that is loaded in the transverse direction, the governing equations of motion can be written as:

$$A_1 \varepsilon^2 \frac{\partial^2 \rho}{\partial z^2} + A_5 \frac{\partial^2 \rho}{\partial x \partial y} + (1-v) A_4 \frac{\partial^2 \rho}{\partial y^2} = 0 \tag{3a}$$

$$A_4 \frac{\partial^2 \rho}{\partial x \partial y} + (1-v) A_5 \frac{\partial^2 \rho}{\partial x^2} + A_1 \frac{\partial^2 \rho}{\partial y^2} = 0 \tag{3b}$$

$$D_s \frac{\partial^2 \mathbf{w}}{\partial z^2} + 2D_s \frac{\partial^2 \mathbf{w}}{\partial x \partial y} + D_s \frac{\partial^2 \mathbf{w}}{\partial y^2} - \rho_s \frac{\partial \mathbf{w}}{\partial t} = f_p \tag{3c}$$

where \( \rho, \mathbf{w}, \mathbf{u} \), and \( \mathbf{w} \) are displacement in the \( x_s, y_s, \) and \( z_s \) direction, respectively, of a point on the mid-surface of the plate considered in the \( x_0-y_0 \) plane. And, the coefficients \( A_i = E h_s/(1-v^2) \) and \( D_s = E h_s^3/(12(1-v^2)) \) correspond to the extensional and bending stiffnesses, respectively, \( \rho_s \) is the density of the plate material, \( h_s \) is the thickness of the plate, \( E \) and \( v \) is Young’s modulus of material and Poisson’s ratio, and \( f_p \) is the distributed transverse load on the plate. The first two equations presented above correspond to the in-plane motion and the last one corresponds to the out-of-plane motion. For the purpose of scaling, only the equation of the out-of-plane motion is considered in Section 2.3. More details of the plate equations are given in Refs. [33,34].
2.3. Scaling laws

Scaling laws are useful to reduce the number of parameters, to clearly identify characteristic properties of the system under consideration, and to indicate which combination of parameters becomes important under a given condition. From the viewpoint of fluid–structure interaction, several dimensionless parameters arise during the non-dimensionalization process of the fluid and structural dynamics equations using a set of suitable reference scales. Depending upon the problem at hand and the type of equations used to model the physical phenomena involved, the resultant set of scaling parameters could vary. In flapping wing flight, three dimensionless parameters related to the fluid dynamics, wing kinematics, and three other dimensionless parameters relevant to the fluid–structure interaction are highlighted next (see Table 1):

(i) The Reynolds number \( (Re) \) represents a ratio between inertial forces and viscous forces.

\[
Re = \frac{\rho \, L_{ref} \, U_{ref}}{\mu},
\]

In flapping wing flight, since lift and thrust are generated by flapping wings, the mean chord length of the wing \( (c_m) \), is used as the reference length \( (L_{ref}) \) and the inverse of the flapping frequency \( (1/f) \) is often utilized as the reference time \( (T_{ref}) \). On the other hand, the reference velocity needs to be selected carefully considering the flight conditions and wing kinematics.

In hovering flight, and the mean wingtip velocity of the flapping wing can be used as the reference velocity, written as \( U_{ref} = U_{tip} = \omega \, R \), where \( \omega \) is the mean angular velocity of the

---

**Fig. 2.** Time histories of the three angles associated with flapping wing motion of hovering flight for (A) a hawkmoth model [42]; (B) a honeybee model [42]; and (C) a fruit fly model [42]. Positional/stroke angle, elevation/deviation angle, and angle of attack of the wing are indicated by solid (green), dash-dotted (blue), and dashed (orange) lines, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Fig. 3.** Schematics diagram for simplified wing kinematics. The stroke plane angle \( (\beta) \) is equal to zero. The global coordinate system \((X,Y,Z)\) is fixed at the center of the stroke plane with \(Z\) direction normal to the stroke plane, the \(Y\) direction perpendicular to the wing axis, and the \(X\) direction parallel to the stroke plane. \( \alpha \) and \( 2h_a/c_m \), are the angle of attack and normalized stroke amplitude with reference to a point of radius of gyration for second moment of the wing [16], respectively.
wing about x-axis ($\omega = 2\Phi f$, where $\Phi$ is the full stroke amplitude, measured in radians) and $R$ is the wing semi-span. Therefore the Reynolds number ($Re$) for hovering flight can be rewritten as

$$Re = \frac{\rho U_{\text{ref}} L_{\text{ref}}}{\mu} = \frac{\rho A_0 \Phi f \omega^2}{\mu}, \quad (5)$$

where the aspect ratio $A_0 = b^2/S$, with the wing planform area ($S$) being the product of the wing span ($b$) and the mean chord length ($c_m$). Note that the Reynolds number is proportional to the flap amplitude, the flapping frequency, square of the mean chord length, and the aspect ratio of the wing (see Table 2).

In forward flight, there are multiple candidates for the reference velocity, for example, the mean wing tip velocity and the forward flight velocity ($U_\infty$). If the reference velocity is chosen as the forward flight velocity, the Reynolds number can be represented as

$$Re = \frac{\rho U_{\text{ref}} U_{\infty}}{\mu} = \frac{\rho c_m U_{\infty}}{\mu} \quad (6)$$

In comparison with the Reynolds number based on the mean wing tip velocity, the Reynolds number based on the forward flight velocity is proportional to the mean chord length and flight velocity, and not related to the flapping frequency and the flapping amplitude.

(ii) The Strouhal number ($St$) describes the relative influence of forward flight ($U_\infty$) versus the flapping speeds [1]. The Strouhal number ($St$) characterizes the vortex dynamics of the wake and shedding behavior of vortices of a flapping wing in forward flight [1,35]. For flapping wing flight, the Strouhal number is defined based on the flapping frequency,
Table 2
Morphological, flight, scaling, and non-dimensional parameters of selected biological flyers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Chalcid Wasp</th>
<th>Fruit fly</th>
<th>Honeybee</th>
<th>Hawkmoth</th>
<th>Rufous Hummingbird</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean chord length: (c_m) (mm)</td>
<td>0.33</td>
<td>0.78</td>
<td>3.0</td>
<td>18.3</td>
<td>12</td>
</tr>
<tr>
<td>Semi-span: (R) (mm)</td>
<td>0.70</td>
<td>2.39</td>
<td>10.0</td>
<td>48.3</td>
<td>54.5</td>
</tr>
<tr>
<td>Aspect ratio: (A_r)</td>
<td>4.24</td>
<td>6.12</td>
<td>6.65</td>
<td>53.3</td>
<td>9</td>
</tr>
<tr>
<td>Total mass: (M) (g)</td>
<td>2.5 (\times)10(^{-7})</td>
<td>0.96 (\times)10(^{-3})</td>
<td>0.1</td>
<td>1.6</td>
<td>3.4</td>
</tr>
<tr>
<td>Flapping frequency: (f) (Hz)</td>
<td>370</td>
<td>218</td>
<td>232.1</td>
<td>261.4</td>
<td>41</td>
</tr>
<tr>
<td>Flapping amplitude: (\Phi) (rad)</td>
<td>2.09</td>
<td>2.44</td>
<td>1.59</td>
<td>2.0</td>
<td>2.02</td>
</tr>
<tr>
<td>Mean wing tip velocity: (U_{tip}) (m/s)</td>
<td>1.08</td>
<td>2.54</td>
<td>7.38</td>
<td>5.04</td>
<td>8.66</td>
</tr>
<tr>
<td>Flight speed: (U_f) (m/s)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Normalized stroke amplitude: (2h_f/\text{ch}_{\text{tip}})</td>
<td>4.4</td>
<td>7.48</td>
<td>5.3</td>
<td>5.28</td>
<td>9.17</td>
</tr>
<tr>
<td>Normalized stroke amplitude: (2h_f/\text{ch}_{\text{ref}})</td>
<td>4.4</td>
<td>4.3</td>
<td>3.05</td>
<td>3.04</td>
<td>5.27</td>
</tr>
<tr>
<td>Reynolds number: (Re)</td>
<td>23</td>
<td>126</td>
<td>1412</td>
<td>5885</td>
<td>6628</td>
</tr>
<tr>
<td>Reduced frequency: (k)</td>
<td>0.355</td>
<td>0.212</td>
<td>0.297</td>
<td>0.296</td>
<td>0.172</td>
</tr>
</tbody>
</table>

The travel distance of full flapping amplitude \((R\Phi)\), and the speed of forward flight, namely,

\[
St = \frac{\mu_{\text{ref}}}{U_{\text{ref}}} = \frac{\int R\Phi}{\int \Phi} = \frac{\mu_{\text{ref}} \Phi}{2U_{\text{ref}}} \tag{7}
\]

This definition offers a measure of propulsive efficiency of flapping wing flyer in forward flight.

(iii) The reduced frequency \((k)\) provides a better measure of unsteadiness associated with a flapping wing than the Strouhal number by comparing spatial wavelength of the flow disturbance with the chord length [35]. It is defined by the angular speed of the flapping wing \((2\pi f)\), mean chord length \((c_m)\), and the reference velocity \((U_{\text{ref}})\), namely,

\[
k = \frac{\pi \mu_{\text{ref}}}{U_{\text{ref}}} \tag{8}
\]

The reduced frequency based on the mean wingtip velocity can be formulated as

\[
k = \frac{\pi \mu_{\text{ref}}}{U_{\text{ref}}} \tag{9}
\]

Note that the reduced frequency is inversely proportional to the flapping amplitude and aspect ratio of the wing and not related to flapping frequency. On the other hand, the reduced frequency based on the forward or cruising flight speed can be rewritten as

\[
k = \frac{\pi \mu_{\text{ref}}}{U_{\text{ref}}} \tag{10}
\]

The reduced frequency is proportional to flapping frequency and the mean chord length, and inversely flight speed. The relationship between the Strouhal number and the reduced frequency based on the forward flight speed is

\[
St = \frac{A_r \Phi}{2 \pi} \tag{11}
\]

(iv) The density ratio \((\rho)\) describes the ratio between the equivalent structural density and fluid density.

\[
\rho = \frac{\rho_s}{\rho_f} \tag{12}
\]

(v) The effective stiffness \((\Pi_1)\) describes the ratio between elastic bending forces and aerodynamic (or fluid dynamic) forces.

\[
\Pi_1 = \frac{Eh_i^3}{2(1-v^2)\rho_f\mu_{\text{ref}}^2 c_m^4} \tag{13}
\]

(vi) If an isotropic shear deformable plate is considered, an additional dimensionless parameter [33,34], the effective rotational inertia \((\Pi_2)\), will be

\[
\Pi_2 = \frac{I_0}{\rho_s c_m^3} \tag{14}
\]

where \(I_0\) is the mass moment of inertia. This describes the ratio between rotational inertia forces and aerodynamic (or fluid dynamic) forces.

As a summary, assuming that the geometric similarity is maintained, the scaling laws for rigid and flexible flapping wing aerodynamics for two sets of reference velocities are listed in Table 1.

Furthermore, in order to understand the effect of the above-scaling parameters on the governing equations, non-dimensional forms of the governing equations are presented based on the reference velocity as follows:

If the flapping wing velocity is chosen as the velocity scale, then the resulting non-dimensional form of the NS equations and the equation of out-of-plane motion of the isotropic plate are

\[
k \frac{c^2 U}{\pi \mu f} + \nabla U - \nabla \sigma + \frac{1}{Re} \nabla^2 U \tag{15}
\]

\[
\Pi_1 \left( \frac{\partial^2 W}{\partial x^2} + 2 \frac{\partial^2 W}{\partial x \partial y} + \frac{\partial^2 W}{\partial y^2} \right) - \frac{\mu}{\rho_s} \left( \frac{k}{\pi} \right)^2 \frac{\partial^2 W}{\partial c^2} = f \tag{16}
\]

where the over-bar designates the dimensionless variable. This form of the equation separates the reduced frequency, the Reynolds number, the density ratio, and the effective stiffness, making it convenient to study the effects of these parameters. For biological flyers, the flapping frequency ranges 10–600 Hz, and the wing length varies from 0.3 to 600 mm yielding the Reynolds number from \(O(10^3)\) to \(O(10^5)\) [1,36]. In this flight regime, unsteady effects, inertia, pressure, internal, and viscous forces are all important.

3. Key attribute of unsteady flapping wing aerodynamics

Natural flyers utilize flapping mechanisms to generate lift and thrust. These mechanisms are related to formation and shedding of the vortices into the flow, varied wing shape, and structural flexibility. Therefore, understanding the vortex dynamics, the vortex–wing interaction, and fluid–structure interaction is very important. A brief introduction to some of the key unsteady mechanisms associated with flapping wings that are frequently encountered in the literature is given next. Specific topics related to the interplay between the
unsteady aerodynamics and kinematics, Reynolds number, and wing geometry will be discussed in detail in Section 4.

3.1. Clap and fling

The earliest unsteady lift generation mechanism to explain how insects fly, found by Weis-Fogh [16], was the clap-and-fling motion of a chalcid wasp, Encarsia formosa. Based on the steady-state approximation, the lift generated by the chalcid wasp was insufficient to stay aloft. To explain this, he observed that a chalcid wasp claps two wings together and then flings them open about the horizontal line of contact to fill the gap with air. During the fling motion, the air around each wing acquires circulation in the correct direction to generate additional lift. A schematic of this procedure is shown in Fig. 5. As illustrated in Fig. 6, Lehmann et al. [37] elucidated this clap-and-fling mechanism with PIV flow visualizations and force measurements using a dynamically scaled robotic wing model. Also numerical investigations further demonstrated lift enhancement due to the clap-and-fling mechanisms at low Reynolds numbers [38–42]. The relative benefit of clap-and-fling lift enhancement strongly depended on stroke kinematics and could potentially increase the performance by reducing the power requirements [43,44]. The clap-and-fling mechanism is beneficial in producing a mean lift coefficient to keep a low weight flyer aloft: numerous natural flyers, such as hawkmoths, butterflies, fruitflies, wasps, and thrips enhance their aerodynamic force production with the clap-and-fling mechanism [16,45–49].

3.2. Rapid pitch rotation

At the end of each stroke, flapping wings can experience rapid pitching rotation, which can enhance the aerodynamic force generation [50]. The phase difference between the translation and the rotation can be utilized as a lift controlling parameter: similar to the Magnus effect, if the wing flips before the stroke ends, then the wing undergoes rapid pitch-up rotation in the correct translational direction enhancing the lift. This is called the advanced rotation. On the other hand, in delayed rotations, if a wing rotates back after the stroke reversal, then when the wing starts to accelerate it pitches down resulting in reduced lift [50]. In a follow-up study Sane and Dickinson [51] related the lift peak at the stroke ends to be proportional to the angular velocity of the wing using the quasi-steady theory. The numerical studies [1,52] showed an increase in the vorticity around the wing due to rapid pitch-up rotation of the wing led to augmentation of the lift generation.

3.3. Wake capture

The wake capture mechanism is often observed during a wing–wake interaction. When the wings reverse their translational direction, the wings meet the wake created during the previous stroke, by which the effective flow velocity increases and additional aerodynamic force peak is generated. Lehmann et al. [37], Dickinson et al. [50], and Birch and Dickinson [53] examined the effect of wake capturing of several simplified fruit fly-like wing kinematics using a dynamically scaled robotic fruit fly wing model at \( Re=1.0–2.0 \times 10^2 \). They compared the force measurement data with the quasi-steady approximation then isolated the aerodynamic influence of the wake. Results demonstrated that wake capture force represented a truly unsteady phenomenon dependent on temporal changes in the distribution and magnitude of vorticity during stroke reversal. The sequence of the wake capture mechanism is illustrated in Fig. 7. Wang [54] and Shyy et al. [1,55] further elucidated the wake capture mechanism and
lift augmentation of the instantaneous lift peak using 2-D numerical simulations. The effectiveness of the wake capture mechanism was a function of wing kinematics and flow structures around the flapping wings [1,37,50,53]. A different view on the effect of wake capture existed as well. Jardin et al. [56] used a NACA0012 airfoil under asymmetric flapping wing kinematics such that in the downstroke the interaction of the previously shed wake with the leading edge vortex (LEV) formation was reduced. In the most cases they considered this reduced effect of wake capture led to a closely attached downstroke LEVs. Compared to a synchronized wing rotation they saw enhanced downstroke aerodynamic loading.

3.4. Delayed stall of leading edge vortex (LEV)

Ellington et al. [57–60] suggested that the delayed stall of LEV can significantly promote lift associated with a flapping wing. The LEV created a region of lower pressure above the wing and hence it would enhance lift. Multiple follow-up investigations [61–64] were conducted for different insect models, resulting in a better understanding on the role played by the LEV and its implications on lift generation. When a flapping wing travels several chord lengths, the flow separates from the leading and trailing edges, as well as at the wing tip, and forms large organized vortices known as a leading edge vortex (LEV), a trailing edge vortex (TEV), and a tip vortex (TiV). In flapping wing flight, the presence of LEVs is essential to delay stall and to augment aerodynamic force production during the translation of the flapping wings as shown in Fig. 8 [18]. Fundamentally, the LEV is generated and sustained from the balance between the pressure-gradient, the centripetal force, and the Coriolis force in the NS equations. The LEV generates a lower pressure area in its core, which results in an increased suction force on the upper surface. Employing 3-D NS computations, Liu and Aono [42] and Shyy and Liu [65] demonstrated that a LEV is a common flow feature in flapping wing aerodynamics at Reynolds numbers O(10^4) and lower, which correspond to the flight regime of insects and flapping wing MAVs. However, main characteristics and implications of the LEV on the lift generation varied with changes in the Reynolds number, the reduced frequency, the Strouhal number, the wing flexibility, and flapping wing kinematics. Milano and Gharib [66] measured the forces generated by pitching rectangular flat plate at approximately Re=4.0 × 10^3 and observed the trajectories yielding maximum average lift based on a genetic algorithm. Results showed the optimal flapping produces LEVs of maximum circulation and that a dynamic formation time that described the vortex formation process of about 4 is associated with production.
of a maximum-circulation vortex [67,68]. Rival et al. [69] investigated experimentally the formation process of LEVs associated with several combinations of pitching and plunging SD7003 airfoils in forward flight using PIV at Re=3.0 \times 10^4. Results suggested that by carefully tuning the airfoil kinematics, thus gradually feeding the LEV over the downstroke, it was to some extent possible to stabilize the LEV without the necessity of a spanwise flow.

Tarascio et al. [70] and Ramasamy and Leishman [71] visualized the flow fields around a biologically inspired flapping wing at Re=1.0 \times 10^4 by a mineral oil fog strobed with a laser sheet and PIV. They presented the presence of a shed dynamic stall vortex that spans across most of the wing span and multiple shedding LEVs on the top surface of the wing during each wing stroke. Also they provided several observations related to the role of turbulence at low Reynolds numbers. Poelma et al. [72] measured the time-dependent 3-D velocity field around a flapping wing at Re=256 based on maximum chord. A dynamically scaled fruitfly wing in mineral oil with hovering kinematics extracted from real insect movements was used. They presented refined 3-D structures of LEVs and suggested including the counter-rotating TEVs to get a complete picture for production of circulation. Lu and Shen [73] highlighted the detailed structures of LEVs for a flapping wing in hover at Re=1.6 \times 10^3. They used phase-lock-based multi-slice digital stereoscopic PIV to show that the spanwise variation along the LEV was time-dependent. Their results demonstrated that the observed LEV systems were a collection of four vortical elements: one primary vortex and three minor vortices, instead of a single conical or tube-like vortex as reported or hypothesized in previous studies [50,57]. Recently, Pick and Lehmann [74] used 3-D three-components multiple-color-plane stereo PIV techniques to obtain a 3-D velocity field around a flapping wing. The need for 3-D PIV is evident since the critical flow features in understanding flapping wing aerodynamics, such as LEVs and unsteady wakes behind an insect body, are inherently 3-D in nature. Compared to the previous findings, they reported similar structure of the LEV but stronger outward axial flow inside the LEV of up to 80% of the maximum in-plane velocity. On the other hand, Lian et al. [75] presented results based on direct numerical simulation to investigate wake structures of hummingbird hovering flight and associated aerodynamic performance. They reported that the amount of lift produced during downstroke is about 2.95 times of that produced in upstroke. Two parallel vortex rings were formed at the end of the upstrokes. There is no obvious leading edge vortex can be observed at the beginning of the upstroke. Although only rigid wing structures were considered, the results were claimed to be in good agreement with PIV measurements of Warrick et al. [76] and Altschuler et al. [77].

3.5. **Tip vortex (TiV)**

Tip vortices (TiVs) associated with fixed finite wings are seen to decrease lift and induce drag [78]. However, in unsteady flows, TiVs can influence the total force exerted on the wing in three ways (see Figs. 8 and 9): (i) creating a low-pressure area near the wing tip [55,79–81], (ii) an interaction between the LEV and the TiV [55,79–81], and (iii) constructing wake structure by downward and radial movement of the root vortex and TiV [71]. First two mechanisms ((i) and (ii)) also were observed for impulsively started flat plates normal to the motion with low aspect ratios: Rivette et al. [82] presented experimentally that the TiVs contributed substantially to the overall plate force by interacting with the LEVs at Re=3.0 \times 10^3. Taira and Colonius [83] utilized the immersed boundary method (IBM) to highlight the 3-D separated flow and vortex dynamics for a number of low aspect ratio flat plates at different angles of attacks. At Re of 3.0 \times 10^3–5.0 \times 10^2. They showed that the TiVs could stabilize the flow and exhibited nonlinear interaction with the shed vortices. Stronger influence of downwash from the TiVs resulted in reduced lift for lower aspect ratio plates.

For flapping motion in hover, however, depending on the specific kinematics, the TiVs could either promote or make little impact on the aerodynamics of a low aspect ratio flapping wing. Shyy et al. [55] demonstrated that for a flat plate with Ar=4 at Re=64 with delayed rotation kinematics, the TiV anchored the vortex shed from the leading edge increasing the lift compared to a 2-D computation under the same kinematics. On the other hand, under different kinematics with small angle of attack and synchronized rotation, the generation of TiVs was small and the aerodynamic loading was well approximated by the analogous 2-D computation. They concluded that the TiVs could either promote or make little impact on the aerodynamics of a low aspect ratio flapping wing by varying the kinematic motions [55].

3.6. **Passive pitching mechanism**

Wing torsional flexibility can allow for a passive pitching motion due to the inertial forces during wing rotation at stroke reversals [84–88]. There were three modes of passive pitching motions which were similar to those suggested by rigid robotic wing model experiments [50]: (1) delayed pitching, (2) synchro-
nized pitching, and (3) advanced pitching. It was found that the ratio of flapping frequency and the natural frequency of the wing were important to determine the modes of passive pitching motions of the wing [88,89]. If the flapping frequency was less than the natural frequency of the wing, then the wing experienced an advanced pitching motion, which led to lift enhancement by intercepting the stronger wake generated during the previous stroke [89]. Moreover, it was shown for 2-D flows, the LEVs produced by the airfoil motion with passive pitching seemed to attach longer on the flexible airfoil than on a rigid airfoil [88].

4. Kinematics, wing geometry, Re, and rigid flapping wing aerodynamics

This section presents a literature survey on flapping wing aerodynamics using experimental, theoretical, and computational approaches. Selected computational efforts of the authors are used to highlight implications of flow structures on the performance of rigid flapping wings. Note that the Reynolds numbers shown for hovering studies in this literature review may differ from those in the referenced studies as consistent definitions (average wing velocity) are used for the sake of comparison in this paper.

Experimentally, numerous previous efforts on flow visualization around biological flyers have been made, including smoke visualizations [47,49,90–92] and PIV [76,77,93–101] measurements. The advance of such technologies has enabled researchers to obtain not only 2-D but also 3-D flow structures around biological flyers [93,98–101] and/or scaled models [72–74] with reasonable resolution in space. At the same time, measurements of wing and body kinematics have been conducted using high-speed cameras [102–110], laser techniques (a scanning projected line method [111], a reflection beam method [112], a fringe shadow method [113], and a projected comb fringe method [114]), and a combination of high-speed cameras and a projected comb-fringe technique with the Landmarks procedure [115]. Advancement in measurement techniques also enabled quantification of flapping wing and body kinematics along with the 3-D deformation of the flapping wing. Recently, data on the instantaneous wing kinematics involving camber along the span, twisting, and flapping motion have been reported (a hovering honeybee [116]; a hovering hover fly and a tethered locust [117,118], a free-flying hawkmoth [119]). These efforts help in establishing more complex and useful computational models [120,121]. Furthermore, the in-vivo measurement of aerodynamic forces generated by biological flyers in free-flight is a very challenging research topic.

Various models have been developed in an effort to understand flapping wing phenomena where the variables are known, controllable, and repeatable. Detailed discussion regarding the experimental and numerical methodologies utilized to examine flapping wing-related studies is beyond the scope of the current effort. Suffice it to say, numerous computational techniques-based moving meshes [122–124] or stationary meshes (cut cell or immersed boundary) [125,126] have been developed. The physical models include NS as well as simplified treatments [127,128]. Some of the experimental methods employed are introduced in the paragraph above.

In the following section recent progress regarding rigid flapping wing aerodynamics is presented. First, studies for forward flight will be described. Then studies for hovering flight will be presented. Explorations on the implications of wing kinematics and wing shape will be touched upon after that. This will be followed by a highlight focusing on the unsteady aerodynamics of 2-D and 3-D hovering flat plates, $Re=O(10^5)$ based on a surrogate modeling approach. Finally, the fluid dynamics related to the LEV, the TEV, and the TiV will be presented, including the authors’ computational efforts to highlight the vortex dynamics of a hovering hawkmoth at $Re=O(10^5)$ and the effect of Reynolds number (size of flyers) on the LEV structures and spanwise flow.

4.1. Single wing in forward flight condition

Von Ellenrieder et al. [129] studied the impact of variation of Strouhal number ($0.2 < St < 0.4$), pitch amplitude ($0^\circ < a_p < 10^\circ$), and phase angle ($65^\circ < \phi < 120^\circ$) between pitching and plunging motion on 3-D flow structures behind a plunging/pitching finite-span NACA0012 wing using dye flow visualization at $Re=164$. The results demonstrated that the variation of these parameters had observable effects on the wake structure. However, they observed a representative pattern of the most commonly seen flow structures and proposed a 3-D model of the vortex structure.
behind a plunging/pitching wing in forward flight. Godoy-Diana et al. [130,131] investigated the vortex dynamics associated with a pitching 2-D teardrop shaped airfoil ($2.2 < z_h < 16.9$, $0.1 < St < 0.5$) in forward flight at $Re=1.2 \times 10^3$ using PIV measurements. Their results illustrated the transition from the von Kármán vortex streets to the reverse von Kármán vortex streets that characterize propulsive wakes. Furthermore, the symmetry breaking of this reverse von Kármán vortex pattern gave rise to an asymmetric wake which was intimately related to the time-averaged aerodynamic force production. Lee et al. [132] numerically investigated aerodynamic characteristics of unsteady force generation by a 2-D pitching and plunging 5% thick elliptic airfoil with inclined stroke plane at $Re=6.8 \times 10^2$. They showed that the thrust was generated due to correct alignment of the vortices at the end of the upstroke and there was a monotonic decrease in thrust as the rotational center of the pitching motion was moved from the leading edge towards the trailing edge ($0.1 < \bar{X} < 0.5$).

Anderson et al. [133] considered harmonically oscillating NACA0012 airfoils in a water tunnel to measure the thrust. After a parametric study ($0 < z_h < 60$, $0.25 < h_f/c_m < 1.0$, $30 < \phi < 110^\circ$) to find the optimum flow condition for the thrust generation ($z_h = 30$, $h_f/c_m = 0.75$, $\phi = 75^\circ$) at $Re=4.0 \times 10^4$ and $St=0.05–0.6$, they proceeded to show the presence of a reverse von Kármán vortex street formed by the vortices shed from the leading and trailing edges for $St$ in range of 0.3 and 0.4 at $Re=1.1 \times 10^5$. Triantafyllou and co-workers [134–136] performed parametric investigations using experiments on the performance of a pitching/plunging NACA0012 airfoil in forward flight at $Re$ between $2.0 \times 10^4$ and $4.0 \times 10^4$, and $St$ between 0.1 and 0.45. Systematic measurements of the fluid loading showed a unique peak efficiency of more than 70% for optimal combinations of the parameters ($e.g.$ $h_f/c = 0.75$, $z_h = 15^\circ$, and $\phi = 90^\circ$) at $St=0.25$ gives an efficiency of 73% and observed that higher thrust can be expected when increasing the Strouhal number and/or the maximum of the angle of attack. Then a parametric range where the efficiency and high thrust conditions were achieved together would fall in the parameter domain they considered. Lai and Platzer [137] used LDV and dye injection techniques to visualize the velocity field and the wake structures of an oscillating NACA0012 airfoil in water at $Re$ ranging from $5.0 \times 10^2$ to $2.1 \times 10^4$. The transition from drag to thrust was seen to depend on the non-dimensional plunging velocity ($kh_h$ which is proportional to $St$), i.e. for $kh_h > 0.4$ the considered airfoil was thrust-producing. Cleaver et al. [138] performed force and supporting PIV measurements on a plunging NACA0012 airfoil at a Reynolds number of $1.0 \times 10^4$, at pre-stall, stall, and post-stall angles of attack. The lift coefficient for pre-stall and stall angles of attack were identified which may also contribute to the selection of the reduced frequency, but also the normalized stroke amplitude. Bohl and Koochesfahani [141] focused on quantifying, via MTV, the vortical structures in the wakes of a sinusoidally pitching NACA0012 airfoil with low pitching amplitude, $z_h=2^\circ$, at $Re=1.3 \times 10^4$. The reduced frequency was set to a relatively high range, between 4.1 and 11.5, to generate thrust. They found that the transverse alignment of the vortices switched at $k=5.7$, i.e. the vortices of positive circulation switched from below to above the vortices of negative circulation. The mean streamwise velocity profile herewith changed from velocity deficit (wake) to velocity excess (jet). However, this switch from the vortex array orientation could not be used to determine the crossover from drag to thrust. Von Ellenrieder and Pothos [142] conducted PIV measurements behind a 2-D plunging NACA0012 airfoil, operating at $St$ between 0.17 and 0.78, and $Re=2.7 \times 10^4$. Their results showed that for Strouhal numbers larger than 0.43, the wake became deflected such that the average velocity profile was asymmetric about the mean heave position of the airfoil. Jones and Babinsky [143] studied the fluid dynamics associated with a three-dimensional 2.5% thick waving flat plate. The spanwise velocity gradient and wing starting and stopping acceleration that exist on an insect-like flapping wing are generated by rotational motion of a finite span wing. The flow development around a waving wing at $Re=6.0 \times 10^4$ was studied using high-speed PIV to capture the unsteady velocity field. Vorticity field computations and a vortex identification scheme reveal the structure of the 3-D flow-field, characterized by strong leading edge and tip vortices. A transient high lift peak approximately 1.5 times the quasi-steady value occurred in the first chord-length of travel, caused by the formation of a strong attached leading edge vortex. This vortex then separated from the leading edge resulting in a sharp drop in lift. As weaker leading edge vortices continued to form and shed lift values recovered to an intermediate value. They also reported that the wing kinematics had only a small effect on the aerodynamic forces produced by the waving wing if the acceleration is sufficiently high. Calderon et al. [144] presented an experimental study on a plunging rectangular wing with aspect ratio of 4, at low Reynolds numbers of $1 \times 10^4$–$3 \times 10^4$. Time-averaged force measurements were presented as a function of non-dimensional frequency, alongside PIV measurements at the mid-span plane. In particular, they focused on the effect of oscillations at low amplitudes and various angles of attack. The presence of multiple peaks in lift was identified for this 3-D wing, thought to be related to the natural shedding frequency of the stationary wing. Wing/vortex and vortex/vortex interactions were identified which may also contribute to the selection of optimal frequencies. Lift enhancement was observed to become more notable with increasing plunging amplitude, to lower reduced frequencies, with increasing angle of attack. Despite the highly 3-D nature of the flow, lift enhancements up to 180% were possible.

Under Research and Technology Organization (RTO) arrangement of the North Atlantic Treaty Organization (NATO) there was...
a community-wide effort organized, which offered a wide range of experimental and computational data for both SD7003 airfoil and flat plate, with kinematics promoting different degrees of flow separation. The detailed information can be found in [145]. Here we present samples of the information collected in this group endeavor. Ol et al. [146] compared PIV flow field measurements of a pitching and plunging SD7003 airfoil at \( Re=6.0 \times 10^4 \), \( k=0.25 \), and \( St=0.08 \) with computed result by Kang et al. [147]. They considered two kinematic motions, a shallow-stall case and a deep-stall case where the maximum effective angle of attack was larger than the former case. In the shallow-stall case where the flow was moderately attached overall, the computed result was able to approximate the flow field measured using PIV well. For the deep-stall case the flow separated just before the middle of the downstroke (i.e. maximum effective angle of attack). The numerical solution showed vortical structures similar to the PIV, but at a later phase of motion. However, the instantaneous lift over a motion cycle obtained from both methods compared well, indicating that the differences in details of the flow structures do not necessarily lead to large differences in the forces integrated over the airfoil, as long as the large-scale flow structures remain similar.

For \( Re=10^6 \) and higher, turbulence influences the development of the flow structures and forces. Ol et al. [145], Baik et al. [148] investigated the fluid physics at \( Re=\Omega(10^6) \) of a pitching and plunging SD7003 airfoil and flat plate, experimentally using PIV focusing on the second-order turbulence statistics. They observed laminar boundary layer and laminar-to-turbulence transition. In a companion paper Kang et al. [147] used RANS computations with SST turbulence model [149] to simulate the same cases [146] to investigate the implications of the turbulence modeling. By limiting the production of turbulence kinetic energy they observed that leading edge separation was dependent on the level of eddy viscosity for the SD7003 airfoil, and hence turbulence, in the flow. Regarding the computed lift, they concluded that the large scale vortical structures in the flow were the contributing factors. Baik et al. [150] conducted an experimental study of a pitching and plunging flat plate at \( Re=1.0 \times 10^4 \) constrained to motions enforcing a pure sinusoidal effective angle of attack. The effect of non-dimensional parameters governing pitching and plunging motion including Strouhal number \( (St) \), reduced frequency \( (k) \), and the plunge amplitude \( (h_a) \) was investigated for the same effective angle of attack kinematics. The formation phase of the LEV was found to be dependent on \( k \): the LEV formation is delayed for higher \( k \) value. It was found that for cases with the same \( k \) the velocity profiles normal to the airfoil surface closely follow each other in all cases independent of pitch rate and pivot point effect. Of course, even though the flow structures with constant \( k \) seemed little affected by Strouhal number and plunging amplitude, the time history of forces along the horizontal (thrust) and normal (lift) directions can be substantially altered because the geometric angle-of-attack, viewed from the ground.

Visbal et al. [151] computed the unsteady transitional flow over a plunging 2-D and 3-D SD7003 airfoil with high reduced frequency \( (k=3.93) \) and low plunge amplitude \( (h_a/C_m=0.05) \) using implicit large Eddy simulations at \( Re=1.0 \times 10^4 \) and \( 4.0 \times 10^4 \). The results showed that the generation of dynamic-stall-like vortices near the leading edge was promoted due to motion-induced high angles of attack and 3-D effects in vortex formation around the wing. Radespiel et al. [152] compared the flow field over a SD7003 airfoil with and without plunging motion at \( Re=6.0 \times 10^4 \). Using a NS solver along with the linear stability analysis to predict transition from laminar-to-turbulent flow, they concluded that transition and turbulence can play an important role in the unsteady fluid dynamics of flapping airfoils and wings at the investigated Reynolds numbers.

### 4.2. Single wing in hovering flight condition

Ellington and co-workers [57–60,103,104] did pioneering research on flapping wing aerodynamics at \( Re=4.0 \times 10^3–7.0 \times 10^3 \). They built a scaled-up robotic hawkmoth wing model and visualized the flow field of hovering hawkmoth wing movements [57–60,103,104] using smoke visualization techniques. They observed that the presence of the LEV at high angle of attack during the downstroke, and suggested that ‘delayed stall’ of the LEV was responsible for high lift production by hovering hawkmoths (see Section 3.4). Dickinson and co-workers [21,22,37,50,51,72,153–159] made original contributions to the understanding the flapping wing aerodynamics at lower Reynolds number regimes \( (Re=1.0 \times 10^2–1.5 \times 10^3) \). They utilized a dynamically scaled robotic fruit fly model wing in an oil tank and conducted systematic experiments to relate the prescribed simplified fly-like kinematics to the resulting aerodynamic forces. They categorized the aerodynamic loading into three parts: forces due to (i) translation (‘delayed stall of the LEV’ [50,72,153–155], (ii) rotation [50,51,156–157], and (iii) interaction with the wakes (‘waste capture’ [50,55]). Furthermore, Fry et al. [158] investigated the aerodynamics of hovering tethered fruit flies using a dynamically scaled robotic fruit fly model at \( Re=1.2 \times 10^2 \). Althshuler et al. [159] studied the aerodynamics of a hovering honeybee using a dynamically scaled robotic honeybee wing model at \( Re=1.0 \times 10^3 \). Their results showed that aerodynamic force enhancement due to wake capturing and rotational forces were important in both fruit fly and honeybee hovering. Sunada et al. [160] measured fluid dynamic forces generated by a bristled wing model with four different wing kinematics using scale-up robotic wings at \( Re=10 \). The results demonstrated that fluid dynamic forces acting on the bristled wing were a little smaller than those on the solid wings.

Nagai et al. [161] used a mechanical bumblebee wing model and measured the resulting forces with strain gauges and flow structures using PIV for a hovering and forward flight at \( Re=\Omega(10^3) \). The comparison between the experimental results and the numerical solutions, computed using a 3-D NS code, showed good agreement quantitatively in forces and qualitatively in flow structures. For the forward flight the relevance of the delayed stall mechanism depended on the advance ratio. They observed that the LEV hardly appeared during upstroke at high advance ratios (over 0.5).

Comparisons of 2-D computational simulations of an elliptic airfoil in hover against 3-D experimental data of the fruit fly model [50] were performed by Wang et al. [162]. They concluded that 2-D computed aerodynamic forces were good approximations of 3-D experiments for the advanced and symmetrical rotation cases considered in their study. Lua et al. [163] experimentally investigated the aerodynamics of a plunging 2-D elliptic airfoil at \( Re \) between \( 6.6 \times 10^2 \) and \( 2.7 \times 10^3 \). The results showed that the fluid inertia and the LEVs played dominant roles in the aerodynamic force generation, and time-resolved force coefficients during plunging motion were found to be more sensitive to changes in pitching angular amplitude than to Reynolds number. Wang [164–166] carried out 2-D numerical investigations on the vortex dynamics associated with a plunging/pitching elliptic airfoil at \( Re=\Omega(10^2)–\Omega(10^4) \). The result showed a downward jet of counter rotating vortices which were formed from LEVs and TEVs. Bos et al. [167] performed 2-D computational studies examining different hovering kinematics: simple harmonic, experimental model [50], realistic fruit fly [158], and modified fruit fly. The results showed that the realistic fruit fly kinematics lead to the optimal mean lift-to-drag ratio compared to other kinematics. Also they concluded that in the case of realistic fruit fly wing kinematics, the angle of attack variation increases the
aerodynamic performance, whereas the deviation levels the forces over the flapping cycle. Kurtulus et al. [168] obtained a flow field over a pitching/plunging NACA0012 airfoil in hover at Re=1.0 × 10^4 experimentally and numerically. A 2-D computation was compared to a pitching and plunging airfoil in water tank. They found that the more energetic vortices which were the most influential flow features on the resulting forces were visible.

Hong and Altman [169,170] investigated experimentally the lift generation from spanwise flow associated with a simple flapping wing at Re between 6.0 × 10^3 and 1.5 × 10^4. The results suggested that the presence of streamwise vortices in the vicinity of the wing tip contributed to the lift on a thin flat plate flapping with zero pitching angle in quiescent air. Isaac et al. [171] used both experimental and numerical methods to investigate the unsteady flow features of a flapping wing at Re between 5.1 × 10^2 and 5.1 × 10^3. They showed a feasible application of the water treading kinematics for hovering using insect/bird-like cambered wings.

4.3. Tandem wing in forward/hovering flight condition

Aerodynamics associated with dragonflies differs from other two-winged insects because forewing and hindwing interactions generate distinct flow features [12]. Sun and Lan [172] studied the lift requirements for a hovering dragonfly using a 3-D NS solver with overset grid methods. They showed that the interaction between the two wings was not strong and reduced the lift compared to single wing configuration, however, large enough to stay aloft. Yamamoto and Isogai [173] conducted a study on the aerodynamics of a hovering dragonfly using a mechanical flapping apparatus with a tandem wing configuration and compared the time history of forces obtained from a 3-D NS solver. The force comparison showed a good agreement and the results suggested that the phase difference between the flapping motions of the fore- and hind-wings only had small influence on the time averaged forces.

Lehmann [44] and Maybury and Lehmann [174] investigated the effect of changing the fore- and hindwing stroke-phase relationship in hover on the aerodynamic performance of each flapping wing by using a dynamically scaled electromechanical insect wing model at Re of approximately 1.0 × 10^3. They measured the aerodynamic forces generated by the wings and visualized flow fields around the wings using PIV. Their results showed that wing phasing determined both mean force production and power expenditures for flight, in particular, hindwing lift production might be varied by a factor of two due to LEV destruction and changes in the strength and the orientation of the local flow vector. Lu et al. [175] showed physical images revealing the flow structures, their evolution, and their interactions during dragonfly hovering using an electromechanical model in water based on the dye flow visualization. Their results showed a delayed development of the LEV in the translational motion of the wing. Furthermore, in most cases, forewing–hindwing interactions were detrimental to the LEVs and were weakened with increase of the wing–root spacing. For a dragonfly in forward flight, the conclusions from Wang and Sun [176] were similar in that the forewing–hindwing interaction was detrimental for the lift, but sufficient to support its weight. They suggested that the downward-induced velocity from each wing would decrease the lift on other wings.

Dong and Liang [177] modeled dragonfly in slow flight by varying the phase difference between the forewing and hindwing and investigated the changes of aerodynamic performance of hindwings. They found that the performance of forewings is not affected by the existence of hindwings; however, hindwings have obvious thrust enhancement and lift reduction due to the existence of forewings. For slow flight, by decreasing the phase angle difference, hindwings will have larger thrust production, slight reduction of lift production, and larger oscillation of force production. Independently, Warkentin and DeLaurier [178] did a series of wind-tunnel tests on an ornithopter configuration consisting of two sets of symmetrically flapping wings of batten-stiffened membrane structures, located one behind the other in tandem. It was discovered that the tandem arrangement can give thrust and efficiency increases over a single set of flapping wings for certain relative phase angles and longitudinal spacing between the wing sets. In particular, close spacing on the order of 1 chord length is generally best, and phase angles of approximately 0° ± 50° give the highest thrusts and propelled efficiencies. Again, referring to other reported studies involving rigid and flexible wings, the aerodynamics and aeroelasticity associated with wing–wing interactions need to be further studied. In another study, Broering et al. [179] numerically studied the aerodynamics of two flapping airfoils in tandem configuration in forward flight at a Reynolds number of 10^4. The relationship between the phase angle and force production was studied over a range of Strouhal numbers and three different phase angles, 0°, 90° and 180°. In general, they found that the lift, thrust and resultant force of the forewing increased compared to those of the single wing. The lift and resultant force of the hindwing was decreased, while the thrust was increased for the 0° phase hindwing and decreased for the 90° and 180° phase hindwings. The lift, thrust and resultant force of the combined fore and hindwings was also compared to the case of two isolated single wings. In general, the 0° phase case did not noticeably change the magnitude of the resultant force, but it inclined the resultant forward due to the decreased lift and increased thrust. The 90° and 180° phase cases significantly decreased the resultant force as well as the lift and thrust. Clearly, more work is needed to help unify our understanding of the wing–wing interactions as function of the individual and relative kinematics and dimensionless flow and structure parameters.

Wang and Russell [180] investigated the role of phase lag between the forewing and the hindwing further by filming a tethered dragonfly and computing the aerodynamic forces and power. They found that the out-of-phase motion in hovering uses almost minimal power to generate sufficient lift to stay aloft and the in-phase motion produce additional force to accelerate in takeoffs. Young et al. [181] investigated aerodynamics of the flapping hindwing of Aeschana juncea dragonfly using 3-D computations at Re between 1.0 × 10^2 and 5.0 × 10^2. The flapping amplitude observed, 34.5°, for the dragonfly maximized the ratio of mean vertical force produced to power required. Zhang and Lu [182] studied a dragonfly gliding and asserted that the forewing–hindwing interaction improved the aerodynamic performance for Re=O(10^2)–O(10^4).

4.4. Implications of wing geometry

Lentink and Gerritsma [183] considered different airfoil shapes numerically to investigate the role of shapes in forward flight on the aerodynamic performance. They computed flow around plunging airfoils at Re of O(10^5) and concluded that the thin airfoil with aft camber outperformed other airfoils including the more conventional airfoil shapes with thick and blunt leading edges. One exception was the plunging N0010 which due to its highest frontal area had good performance. Usherwood and Ellington [184] examined experimentally the effect of detailed shapes of a revolving wing with planform based on hawkmoth wings at Re=5.0 × 10^3. The results showed that detailed leading
edge shapes, twist, and camber did not have substantial influence on the aerodynamic performance. In a companion paper Usherwood and Ellington [185] examined experimentally the effect of aspect ratio of a revolving wing with the same hawkmoth planform [184] adjusted to aspect ratios ranging from 4.53 to 15.84 with corresponding $Re$ of $1.1 \times 10^3$–$2.6 \times 10^4$. The results showed the influence of the aspect ratio was relatively minor, especially at angle of attack below $50^\circ$. Luo and Sun [186] investigated numerically the effects of corrugation and wing planform (shape and aspect ratio) on the aerodynamic force production of model insect wings in sweeping motion at $Re=2.0 \times 10^2$ and $3.5 \times 10^3$ at angle of attack of $40^\circ$. The results showed that the variation of the wing shape almost unaffected the force generation and the effect of aspect ratio was also remarkably small. Moreover, the effects of corrugated wing sections in forward flight were studied in numerical simulations [187,188] and a numerical–experimental approach [189]. The results demonstrated that the pleated airfoil produced comparable and at times higher lift than the profiled airfoil, with a drag comparable to that of its profiled counterpart [187].

Altschuler et al. [190] tested experimentally the effect of wing shape (i.e. with sharpened leading edges and with substantial camber) of a revolving wing at $Re$ between $5.0 \times 10^3$ and $2.0 \times 10^4$. Their results demonstrated that lift tended to increase as wing models become more realistic as did the lift-to-drag ratios Ansari et al. [191] used an inviscid model for hovering flapping wings to show that increasing the aspect ratio, wing length, and wing area enhances lift. Furthermore, they suggested that for a flapping wing MAV, the best design configuration would have high aspect ratio, straight leading edge, and large wing area outboard. The pitching axis would then be located near the center of the area in the chordwise direction to provide the best compromise for shedding vortices from the leading and trailing edges during the stroke reversal. Green and Smith [192] examined experimentally the effect of aspect ratio and pitching amplitude of a pitching flat plate in forward flight at $Re$ between $3.5 \times 10^3$ and $4.3 \times 10^3$ and aspect ratios of 0.54 and 2.25. They measured unsteady pressure distributions on the wing, and compared to the PIV measurements of the same setup [193]. They concluded that the 3-D effects increased with decreasing aspect ratio, or when the pitching amplitude increased.

Kang et al. [147] investigated the airfoil shape effect at $Re=O(10^4)$ by comparing the flow field around pitching and plunging SD7003 airfoil and flat plate using PIV [148], and CFD in forward flight. It was observed that for the flat plate the flow was not able to make turn around the sharper leading edge of the flat plate and eventually separated at all phases of motion. The flow separation led to larger vortical structures on the suction side of the flat plate hence increasing the area of lower pressure distribution on the flat plate surface. From the time history of lift, available for the CFD, it was seen that for the flat plate these vortical structures increased the lift generation compared to the SD7003, which had a blunter leading edge.

4.5. Implications of wing kinematics

A primary driver of the unsteady aerodynamics in flapping wing flight is the wing motions. Yates [194] suggested that the choice of the position of the pitching axis may enhance performance and control of rapid maneuvers and thus enable the organism to more adeptly cope with turbulent environmental conditions, to avoid danger, or to more easily capture food. The instantaneous fluid forces, torques, and rate of work done by the propulsive appendages were computed using 2-D unsteady aerodynamic theories. For a prescribed motion in forward flight, the moment and power were further analyzed to find the axes, for which the mean square moment was minimal, the mean power to maintain the moment was zero, the mean square power to maintain moment was a minimum, and the mean square power to maintain lift equaled the mean square power. Sane and Dickinson [51] investigated the effect of location of pitching axis on force generation of the flapping fruit fly-like wing in the first stroke using a scaled-up robotic wing model. They estimated the rotational forces based on the quasi-steady treatment and blade element theory. The results showed that rotational forces decrease uniformly as the axis of rotation moves from the leading edge towards the trailing edge and change sign at approximately three-fourths of a chord length from the leading edge of the wing [50].

Ansari et al. [195] studied the effects of wing kinematics on the aerodynamic performances of insect-like flapping wings in hover based on non-linear unsteady aerodynamic models. They found that the lift and the drag increased with increasing flapping frequency, stroke amplitude, and advanced wing rotation. However, such increases were limited by practical considerations. Furthermore, the authors mentioned that variations in wing kinematics were more difficult to implement mechanically than variations in wing planform. Hsieh et al. [196] investigated the aerodynamics associated with the advanced, delayed, and symmetric rotation and decomposed the lift coefficients in terms of the lift caused by vorticity, wing velocity, and wing acceleration. The results suggested that while the symmetric rotation had the most lift due to vorticity on the surface of the wing and in the flow, the maximum total lift is found with advanced rotation. Oyama et al. [197] optimized for the mean lift, mean drag, and mean required power generated by a pitching/plunging NACA0012 airfoil in forward flight using a 2-D NS solver at $Re=10^5$. The multi-objective evolutionary algorithm was used to find the pareto front of the objective functions (i.e. by considering the reduced frequency, plunge amplitude, pitch amplitude, pitch offset, and phase shift as the design variables). They found that the pitching angle amplitude (between $35^\circ$ and $45^\circ$) was optimum for high-performance flapping motion and the phase angle between pitch and plunge of about $90^\circ$. In addition, the reduced frequency was a tradeoff parameter between minimization of required power and maximization of lift or thrust where smaller frequency leads to smaller required power.

4.6. Surrogate modeling for hovering wing aerodynamics

Numerous studies, including Wang et al. [162] and references cited above, reported similarities as well as differences in aerodynamics and flow structures between 2-D and 3-D flapping wings. There is a need for establishing a comprehensive framework to address these matters. From a vehicle development perspective, since 3-D Navier–Stokes simulations are expensive to run, if 2-D simplifications can adequately reproduce the main aerodynamic features of 3-D flapping wing, naturally, the needed data can be generated much more economically. Trizila et al. [198,199] used surrogate modeling techniques [200–202] to investigate a large number of hovering wing cases to develop surrogate models for time averaged lift and thrust. They identified regions where 2-D and 3-D results (time averaged lift and thrust) were comparable, as well as those that were substantially different. Furthermore, based on the guidance from the surrogate models, they probed the fluid physics associated with 2-D and 3-D cases, and were able to highlight the roles played by the LEV and TiVs in unsteady aerodynamics. The impact on lift from the 2-D/3-D unsteady mechanisms is detailed further in the subsequent sections.
The flapping wing scenario is simplified to better identify and understand the competing interactions and influences. The wing is modeled as a flat plate with 2% thickness with respect to the chord; for the finite wing, the aspect ratio is 4. The mid-span cross-section of the 3-D model is used for the 2-D simulation. Simplified wing kinematics are employed (see Section 2.1) and the mid-chord of the rigid airfoil is selected as the pitching axis. It was shown the position of the axis about which the wing pitches will influence the aerodynamics [50, 51, 194, 195, 197] and Section 4.5. The results shown will not be independent of this choice of rotation (which is beyond the scope of the study presented), however, the understanding of the most prominent features will be relevant to flapping flight in general. The three kinematic parameters (or the design variables in surrogate modeling), \( h_a \) (plunge amplitude), \( z_a \) (pitch amplitude), and \( f \) (phase lag between pitching and plunging motion) can be varied independently.

The range of the design variables is as follows. The normalized stroke amplitude is representative of a range of flapping wing flyers: \( 2h_a/c_{mg} < 4.0 \). Details on pitch amplitude and phase lag are not as plentiful in the literature, so cases are chosen with low angles of attack (AoA\( \min=10^\circ \); high pitching amplitude) and high angles of attack (AoA\( \min=45^\circ \); low pitching amplitude): \( 45^\circ < z_a < 80^\circ \). The bounds on phase lag are chosen symmetric about the synchronized hovering: \( 60^\circ < f < 120^\circ \). Although delayed rotation is not a focus of many studies found in literature, due to its reduced ability to generate lift compared to its normal hovering and advanced rotation brethren in 2-D flow, it will be shown that the interaction of a TiV with a LEV for a delayed rotation case can exhibit prominent 3-D effects beneficial to the wing’s lift performance.

A Navier–Stokes solver [203] is used to obtain the time-dependent flow solution. Due to the kinematic constraints there are only two relevant non-dimensional groups in the incompressible case. The plunging amplitude to chord ratio, \( 2h_a/c_{mg} \), and the Reynolds number: since \( Re \) is being held constant (for a fixed medium) the plunging amplitude and flapping frequency are not independent. The Reynolds number, based on the average tip velocity, is equal to 65, which is similar to the fruit fly, *Drosophila melanogaster*. The reduced frequency, \( k \), contains the same information as the normalized stroke amplitude (see Section 2.1).

The surrogate modeling tools [200–202], trained with CFD data (order of days to obtain the results for one training point), are used to efficiently evaluate off design points (in a fraction of a second), global trends across the design space, kinematic variables’ sensitivity, and tradeoffs between lift and power. The quantities of interest, the objective functions in the surrogate modeling nomenclature, are mean lift coefficient, \( C_L \), and an approximation of the power required over the stroke cycle. For more details on the implementation and error quantification the reader is referred to Refs. [198,199] (Fig. 10).

Fig. 11 shows iso-surfaces of the mean lift coefficient where each axis corresponds to one of the kinematic parameters, i.e. \( h_a \), \( z_a \), or \( f \). Fig. 11 (A), (B), and (C) correspond to the 2-D lift, 3-D lift, and...
and difference between the two, respectively. Observations that are immediately apparent are that kinematic combinations with low \( \alpha_a \) (high AoA) and advanced rotation (high \( \phi \)) have the highest mean lift in 2-D and 3-D. This result qualitatively agrees with the results of Wang et al. [162] and Sane and Dickinson [51]. Due to the non-monotonic response in phase lag found in the 2-D lift response at high pitching amplitudes (see Fig. 11 (A)), there are two regions where there is low, and possibly negative, mean lift generation. The first region is defined by low plunge amplitudes (low \( 2h_a/c_m \)), low AoA (high \( \alpha_a \)), and advanced rotation (high \( \phi \)). The second region is defined by low AoA (high \( \alpha_a \)), and delayed rotation (low \( \phi \)) and is where the 3-D kinematics also generate low mean lift values.

A variable's sensitivity is directly related to the gradients along the respective design variables, while a more quantitative measure is the global sensitivity analysis; these measures are examined in Trizila et al. [198,199]. It is seen that the gradients along the \( \alpha_a \) and \( \phi \) axes are much more significant than that along the normalized stroke amplitude (\( 2h_a/c_m \)). (Note: Lua et al. [163] find the effect of \( Re \) is noticeably smaller than that of \( \alpha_a \) on the mean lift.) With these observations, the trends seen as a variable is varied are more clearly illuminated, but also the associated limitations are just as apparent. For example, advancing the phase lag is beneficial in 2-D except when at high \( \alpha_a \); in 3-D there is no such exception within the bounds studied. Comparisons with Sane and Dickinson’s experiments [51] show qualitative agreement in the trends in mean lift as a function of \( \alpha_a \) and \( \phi \) within the common ranges, with the noticeable difference in setups being the current study using pure translation to represent the plunge whereas the experimental study flaps about a pivot point.

The difference between 2-D and 3-D lift may raise the question about areas of the design space for which 2-D computations may sufficiently approximate their analogous 3-D counterparts or where there are substantial 3-D effects which would preclude such a comparison. In order to illustrate comparable 2-D and 3-D cases, a training point for which the full CFD solution was chosen from the region showing similar mean lift. Fig. 12 presents the time history of lift for a synchronized rotation

![Image](image-url)
case with generally low AoA ($2h_s/c_{mrg} = 3.0$, $\alpha_s = 80^\circ$, $\phi = 90^\circ$). The figure shows not only good agreement for this case in the mean or time averaged sense, which can be deduced from the surrogate models, but also instantaneously. Fig. 13 shows little variation in the spanwise vortex dynamics encountered by each wing cross-section. This is accomplished by way of an iso-surface of the quantity $Q$, a measure of rotation which separates out the shear and is defined in [204], colored by the spanwise vorticity. In this configuration the LEV will be red/yellow, the TEV blue, and the TiV green. Next to the flow field shots are plots of the spanwise lift at the selected time instances which also show decent agreement between 2-D and 3-D. The variation that is present is generally confined locally to the tips, though the magnitude is small. The TiVs are most prominent at the end of translation, but as seen from the spanwise distribution of lift, are limited in influence. Note that other 2-D and 3-D cases agreed in the mean sense, but instantaneously differed and this situation is explored further in Trizila et al. [198,199].

In contrast to the case illustrated above, another set of kinematics chosen for which the 2-D and 3-D surrogate models did not exhibit such agreement. Fig. 14 illustrates a delayed rotation case with high AoA ($2h_s/c_{mrg} = 2.0$, $\alpha_s = 45^\circ$, $\phi = 60^\circ$), where the 2-D and 3-D cases show noticeable differences in the instantaneous lift. The source of these differences is better illustrated in Fig. 15. The spanwise variation in the vortex dynamics is seen in the plots of the flow field as well as in the lift distribution across the span of the wing. Once again the iso-surface connected to the wing can be distinguished by the LEV (red and yellow), the TEV (blue) and the TiV (green). The TiV has noticeable impact on the resulting flow features and aerodynamic loading. First, the distribution of lift due to pressure shows a local peak at the tips which is a force enhancement over the 2-D counterpart. There also appears to be an LEV anchoring mechanism, which keeps the LEV attached over a portion of the span near the wing tip thereby increasing the lift farther inboard. The net effect of the tip vortices is a substantial increase in performance for this set of kinematics.

In Trizila et al. [198,199] the design space was more thoroughly explored. A conclusion that is not apparent from the steady-state aerodynamics is that TiVs can be utilized to increase performance in the context of low Re flapping wing flight. Due to the competing effects the TiV introduce, the role the TiV plays can vary from case to case (i.e. its mere presence does not guarantee a performance increase). The TiV can introduce a low-pressure region on the upper wing surface and anchor the LEV, which would have otherwise been shed. On the other hand, it can also induce downwash thereby reducing the effective angle of attack and consequent lift.

4.7. Unsteady flow structures around hawkmoth-like model in hover

There are a number of computational studies of realistic wing configurations of a hornet [205], a bumblebee [206,207], a hawkmoth [42,61–64,79,208], a honeybee [42], a drone fly [209], a hover fly [210], a fruit fly [42,52,80,81,211,212], and a thrips [42] accompanied by appropriate wing kinematics. Moreover, references cited throughout the literature review have elucidated aspects of the unsteady vortical flow phenomena by way of dynamically scaled robotic wings and biological flyers. In order to expand our knowledge about the LEV mechanisms and unsteady 3-D flow features associated with a biological flyer-like flapping model, a numerical study on the aerodynamics and vortex dynamics around a realistic hawkmoth model with complex flapping wing motions is discussed and highlighted in this subsection.

Figs. 16 and 17 show the morphological and wing kinematics models of a realistic hawkmoth model. Computations are performed using “a biology-inspired dynamic flight simulator [81,213]”. This framework is capable of simulating an insect with realistic wing–body morphologies and flapping-wing/body kinematics. Detailed descriptions of the computational modeling can be referred to in Refs. [64,79,208,213].

4.7.1. Vortex dynamics of hovering hawkmoth

Iso-vorticity-magnitude surfaces around a hovering hawkmoth at nine-time instants in a flapping cycle are shown in Fig. 18. The contour color on the iso-vorticity-magnitude surfaces in Fig. 18 indicates the magnitude of the normalized helicity density, which is computed by projecting the spin vector of a fluid element onto its momentum vector, being positive (red) if these two vectors point in the same direction and negative (blue) if they point to the opposite direction. This quantity can be useful for illustrating helical vortex structures. One aim of the study is to correlate the time histories of the aerodynamic loadings with the dominant flow features [214]. As such, the time histories of the aerodynamic forces on the wings and body are plotted in Fig. 19: (A) vertical (lift) force, and (B) horizontal (drag or thrust) force, respectively.

4.7.1.1. Flow structures during downstroke

In the first half of the downstroke (see Fig. 17), Fig. 18 A–1, a horseshoe-shaped vortex is generated by the initial wing motion of downstroke. Poelma et al. [72] showed a similar three-dimensional flow structure around an impulsively started dynamically scaled flapping wing using PIV. The horseshoe-shaped vortex is composed of three vortices, namely, a LEV, a TEV, and a TiV, and it grows in size as the translational and angular velocity of the wing increases. In particular, the radii of the LEVs and the TEVs expand from the wing root to the wing tip, resulting in a 3-D vortex structure. These vortices produce a low-pressure region in their core and on the upper surfaces of the wing (Fig. 20 (1)) when they are attached. Lift forces shows a peak at the corresponding time instant (Fig. 19 (A) and Fig. 17 (b)). Therefore, this would imply that the LEVs, the
TEVs, and the TiVs enhance the lift force generation in hovering flight of the hawkmoth. It is seen that the TEVs generated by the right and left wings meet at the rear body (Fig. 18 A-2), resulting in an interaction with each other. These vortex–vortex and vortex–body interactions lead to more complex vortical features around the hovering hawkmoth than the 2-D flow structures seen in Fig. 13.
During most of the downstroke, the doughnut-shaped vortex ring pair has an intense, downward jet-flow close to the wing tip, the vortical structure is twisted (rolled-up) behind each wing, which results from an interaction between the broken-down LEV and the shed TV (Fig. 18 A-4). This helps to illustrate the rich complexity of 3-D interactions experienced in flapping wing flight.

Continuing on, during the second half of the downstroke (see Fig. 17 (d)), the TVVs enlarge gradually. Eventually, when the wings approach the end of the downstroke, the LEVs and the TVVs weaken and begin to detach from the wings. An additional vortex is observed around the wing root and connects with the shed TEV (Fig. 18 A-5). The doughnut-shaped vortex rings of the wing pair break up into two circular vortex rings, eventually forming a doughnut-shaped vortex ring close to the wing tip, the vortical structure is twisted (rolled-up) behind each wing, which results from an interaction between the broken-down LEV and the shed TV (Fig. 18 A-4). This helps to illustrate the rich complexity of 3-D interactions experienced in flapping wing flight.

In the first half of the supination (see Fig. 17 (e)), occurring near the end of the downstroke, as the flapping wing slows down, the attached vortices (the LEVs and the TVVs) are shed from the wings. At this time instant, a pair of downstroke stopping vortices is observed wrapping around the two wings (Fig. 18 B-1). When the flapping wings begin to pitch quickly about the spanwise axis, a pair of upstroke starting vortices is detected around the wing tip and the trailing edge (Fig. 18 B-2).

4.7.1.2. Flow structures during upstroke. In the second half of the supination (Fig. 17 (f)), TVVs and TiVs associated with the beginning of the upstroke are generated when the flapping wings accelerate rapidly. The downstroke wakes of the circular vortex rings are subsequently captured (Fig. 18 B-3), but a correlation with the time history of lift shows a relatively minor impact. The idea of wake capture has been documented many times [1,36,43,44,50,55,56] and Section 3.4. The point here is that the role it plays in the lift generation can vary depending on the context. Dickinson et al. [50] experimented with 3-D flapping kinematics at lower Re, i.e. O(10^3), which showed a significant contribution to lift from the wake capturing. In contrast, in the present study, the hovering hawkmoth-like wing kinematics at a Re of 6.3 x 10^5 do not show a prominent wake capturing mechanism in the lift histories. Another study [81] looking at fruit fly wing motions at Re O(10^5) also demonstrated a situation where the wake capturing had little impact on the aerodynamic loading. While still in the first half of the upstroke (Fig. 17 (g)), when the flapping wings begin to accelerate upward, the TVVs and TiVs are shed from the two wings (Fig. 18 C-1). The TVVs are continually regenerated and fed while the LEV generation is initiated. Together with the TVVs, the LEVs and TiVs form a horseshoe-shaped vortex pair wrapping each wing. Similar to what is seen during the downstroke, the horseshoe-shaped vortex grows and evolves into a doughnut-shaped vortex ring for each wing. Root vortices are also detected at this stage and join the vortex ring as illustrated in Fig. 18 C-2. The difficulty in clearly demarcating and expressing the relevant phenomena is a direct consequence of the 3-D nature of the vortex structures and their subsequent interactions. Note that due to asymmetric variation of angle of attack between the upstroke and downstroke, the LEVs generated in the upstrokes are smaller than those in the downstrokes. In the second half of the upstroke (Fig. 17 (h)), the doughnut-shaped vortex rings elongate and deform while maintaining their ring-like shape (Fig. 18 C-3). During most of the upstroke, the downwash induced through the hole of each vortex ring is observed to be similar to that of the downstroke, despite the asymmetric forward and backstroke kinematics.

During early pronation (Fig. 17 (i)) at the end of the upstroke, the attachment points of the shed TVV move slightly (approximately 10–20% of the wing length) from the wing tip to the wing root during the course of the upstroke due to the wing movement (Fig. 18 D-1). Concurrently, the stopping vortices are observed wrapping around each wing (Fig. 18 D-1). As the wings begin to rotate quickly, the upstroke stopping vortices are shed from the trailing edge and the downstroke starting vortices are detected at the leading edge and wing tip. Thereafter, the upstroke doughnut-shaped vortex rings are shed downward, breaking up into two vortex wake rings; the root vortices are also shed, forming a single vortex ring under the body (Fig. 18 D-2). During late pronation (Fig. 17 (a)), at the end of the upstroke and beginning of the downstroke, starting vortices are observed around the leading and trailing edges and wing tip (Fig. 18 D-3).

4.7.1.3. Flight energetics. As to the aerodynamic force generation, two peaks in the lift force are predicted during each stroke for a hovering hawkmoth (see Fig. 19). Considering the correlation between the aerodynamic forces and the key unsteady flow features associated with flapping wings discussed in Section 3, the
delayed stall of the LEV and contributions from the TEV and TiV are responsible for the first lift peak. The second peak is likely to be associated with a contribution from the rapid increase in vorticity [1,52] as the wing experiences a fast pitching motion. Relevant to estimates of energy consumption in natural flyers and control applications in MAV design/construction, the computed time histories of the aerodynamic torques are plotted in Fig. 21 (A) rolling, (B) pitching and (C) yawing, respectively. Note that the term 'total wing' in Fig. 21 is the sum of the torques of the right and left wings. Two of the aerodynamic torques are relatively comparable, the aerodynamic rolling (ART) and yawing (AYT) torques of the two wings ('total wing') are much smaller than those of the aerodynamic pitching torque (APT). The time-varying total ART and total AYT are mostly zero over a flapping cycle because of the symmetrical flapping motion of the left and right wings and therefore the averaged total ART and AYT become approximately zero (Fig. 21 (A) and (C)). On the other hand, the APT of the two wings varies over a range, from \(-0.6 \times 10^{-3}\) to

![Fig. 15](image-url). The lift per unit span (right column) and iso-Q surfaces (Q=0.75) (left column) colored by spanwise vorticity over half of the wing using the kinematic parameters, \(2h_s/c_{ms} = 2.0, z_a = 45^\circ, \phi = 60^\circ\) at \(t/T = 0.05, 0.25\) and 0.45. Note that solid and dashed lines in (A-2) and (B-2) indicate the results from the three- and two-dimensional wings, respectively. The spanwise variation in forces is examined with the 2-D equivalent (redline) marked for reference. The TiV leads to increased lift in their immediate region as well as anchor the vortex shed from the leading edge. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

\(Q\)-criterion

<table>
<thead>
<tr>
<th>(t/T = 0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{L+}) per unit length</td>
</tr>
<tr>
<td>(-3)</td>
</tr>
<tr>
<td>(Y)</td>
</tr>
</tbody>
</table>

\(t/T = 0.25\)

| \(t/T = 0.45\)
0.4 × 10^{-3} \text{Nm} in a flapping cycle. Note that a consequence of negative APT pulls the body orientation of the insect nose-down while a positive APT is associated with nose-up. The transient APT of the two wings has four local maxima, at early/late pronation/supination. In short, the prediction of the APT of the wings shows that the body may experience a nose-down pitching torque during: peak (1) late pronation during the first half of the downstroke, peak (2) the start of supination during the late downstroke, and peak (4) the start of pronation during the upstroke. Based on the computed transient aerodynamic forces and the velocity of the flapping wings, the muscle-mass-specific aerodynamic power $P_{\text{aero}}$ can be calculated:

$$P_{\text{aero}} = \frac{F_{\text{aero}} \cdot V_{\text{wing}}}{M_m}$$

(17)

where the mass of flight muscle, $M_m$ is assumed to contribute up to 23% of the total body mass of the hawkmoth (1.6 g), $F_{\text{aero}}$ is the aerodynamic force, and $V_{\text{wing}}$ is the velocity vector of the flapping wing. The power necessary to overcome the air resistance is denoted $P_{\text{aero}}$. The time histories of $P_{\text{aero}}$ for the wings are plotted in Fig. 22. The maxima of the $P_{\text{aero}}$ coincide with the peaks experienced in the transient aerodynamic forces. The computed mean $P_{\text{aero}}$ (87.2 W kg$^{-1}$) is very close to the experimental estimation by Willmott and Ellington [104].

4.8. Effect of the Reynolds number on the LEV structure and spanwise flow

As discussed previously, the lift enhancement due to the delayed stall of the LEV is important in flapping wing flight [1,50,57] and Section 3.4. The formation of the LEV depends on the wing kinematics, the details of wing geometry, and the Reynolds number [1,42,65,155,213]. Liu and Aono [42] investigated the interaction between LEV, TiV, and vortex ring structures. Tang et al. [215] investigated numerically the effects of Reynolds number on 2-D hovering airfoil aerodynamics at $Re=7.5 \times 10^{3}-1.7 \times 10^{4}$. They showed that in low Reynolds number regimes, $O(10^2)$, the viscosity dissipated the vortex structures quickly and led to essentially symmetric flow structures and aerodynamic forces between the forward and backward stroke while at higher Reynolds numbers, the history effect was influential, resulting in distinctly asymmetric phenomena between strokes. In order to examine the Re effect on LEV structures and spanwise flow for realistic wing body configurations with appropriate kinematic motions, a numerical investigation is presented next: realistic models of hawkmoth ($Re=6.3 \times 10^{5}$, $k=0.30$), honeybee ($Re=1.1 \times 10^{3}$, $k=0.24$), fruitfly ($Re=1.3 \times 10^{5}$, $k=0.21$), and thrips ($Re=1.2 \times 10^{3}$, $k=0.25$) in hover considering different representative kinematic parameters (flap amplitude, flap frequency, and type of prescribed actuation) and dimensionless numbers (Reynolds number, reduced frequency) in each case [42]. The morphological and kinematical model of a hawkmoth is shown in Figs. 16 and 17. For other insect models, similar information and computational models can be obtained in Refs. [42,213].

Fig. 23 shows the computed velocity vector distributions on an end-view plane at 60% semi-span for these four insects. It may be noted that the LEV structure and the spanwise flow in the hawkmoth and the fruitfly cases (see Fig. 23 (A) and (C)) are in good qualitative agreement with the corresponding experimental results reported in Ref. [155]. For the thrips flight ($Re=1.2 \times 10^{3}$, $k=0.25$), the LEV forms upstream of the leading edge and spanwise flow is weakest among all cases. For the fruit fly case ($Re=1.3 \times 10^{2}$, $k=0.21$), the structure of the LEV is smaller than the hawkmoth and the honeybee cases. The fruit fly LEV is tube-like and ordered, and spanwise flow is observed around the upper region of trailing edge. The hawkmoth ($Re=6.3 \times 10^{3}$, $k=0.30$) and honeybee ($Re=1.1 \times 10^{3}$, $k=0.24$) cases yield much more pronounced spanwise flow inside the LEV and upper surface of the wing, which together with the LEV forms a helical flow structure near the leading edge (see Figs. 23 (A–1) and (B–1)). Fig. 24 shows the spanwise pressure-gradient contours on the wing of four typical insects during the middle of the downstroke. Compared to the hawkmoth and the honeybee, even though the wing kinematics and the wing–body geometries are different, fruit flies, at a Re of $1.0 \times 10^{2}$–2.5 $\times 10^{2}$, cannot create as steep pressure-gradients at the vortex core; nevertheless, they seem to be able to maintain a stable LEV during most of the down and upstroke. While the LEVs on both wings of hawkmoth and honeybee experience a break-down near the middle of the downstroke, the LEV on the fruit fly’s wing remains attached during the entire stroke, eventually breaking down during the subsequent supination or pronation [81].

To illustrate the Reynolds number effect, ceteris paribus, numerical results were obtained for a single wing–body geometry and kinematics combination [65]. Qualitative similar flow structures around the flyer were observed. This implied that even
though each flapping flyer had a unique wing and body kinematics and morphologies, the Reynolds number was the dominant parameter dictating the LEV structure and the spanwise flow.

The helicopter blade model has been used to help explain the flapping wing aerodynamics; however, spanwise axial flows are generally considered playing a minor role in influencing the helicopter aerodynamics [216,217]. In particular, the helicopter blades operate at a substantially higher Reynolds number and lower angle of attack. The much larger aspect ratio of a blade also makes the LEV harder to anchor. These are key differences between the helicopter blades and the typical biological wings. Usherwood and Ellington [184] investigated the aerodynamic performance associated with ‘propeller-like’ rotation of the hawkmoth wing model ($Re=O(10^3)$). Their results reported that revolving models produced high lift and drag forces because of the presence of the LEV. Knowles et al. [218] performed computational studies involving translating 2-D and rotating 3-D flat plate models at $Re=5.0 \times 10^2$. Their results presented, that for 2-D flows, the LEV was unstable but the rotating 3-D flat plate model at high angle of attack produced a conical LEV, as it was observed in the results presented earlier and by others [184,219]. Moreover, they mentioned that if the Reynolds number increased above a critical value, a Kelvin–Helmholtz instability [220] occurred in the LEV sheet, resulting in the sheet breaking down on outboard sections of the wing. Recently, Lentink and Dickinson [221] revisited existing hypotheses regarding stabilizing the LEV. They systematically investigated the effects of propeller-like motion of a fruit fly wing model on the aerodynamic performance and the LEV stability at $Re=1.1 \times 10^2–1.4 \times 10^4$ based on theoretical and experimental approaches. They stated that the LEV was stabilized by the “quasi-steady” centripetal and Coriolis accelerations that were present at low Rossby numbers (i.e. a half of the aspect ratio) and resulted from the propeller-like sweep of the wing. In addition, they suggested that the force augmentation through a stably attached LEV could represent a convergent solution for the generation of high fluid forces over a range of Reynolds numbers. From the viewpoint of unsteady aerodynamics, the LEV as a lift enhancement mechanism at higher $Re$ range ($O(10^5–10^6)$) may be questionable because a dynamic-stall

Fig. 18. Visualization of flow fields around a hovering hawkmoth. Iso-vorticity-magnitude surfaces around a hovering hawkmoth during (A) the downstroke, (B) the supination, (C) the upstroke, and (D) the pronation, respectively. The color of iso-vorticity-magnitude surfaces indicates the normalized helicity density which is defined as the projection of a fluid’s spin vector in the direction of its momentum vector, being positive (red) if these two vectors point in the same direction and negative (blue) if they point in the opposite direction. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
vortex on an oscillating airfoil is often found to break away and to convect elsewhere as soon as the airfoil translates [222].

5. Flapping wing aeroelasticity

The topic of aeroelasticity in flapping wings has recently increased in activity though a full picture of the basic aeroelastic phenomena is still not clear. Analytical investigations by Daniel and Combes [223] suggested that aerodynamic loads were relatively unimportant in determining the bending patterns in oscillating wings. Subsequently, experimental investigations by Combes [224], and Combes and Daniel [225] found that the overall bending patterns of a Hawkmoth wing were quite similar when flapped (single degree-of-freedom flap rotation) in air and helium, despite a 85% reduction in fluid density in the latter, suggesting that the contribution of aerodynamic forces was relatively small compared to the contribution of inertial–elastic forces during flapping motion. However, they mentioned that realistic wing kinematics might include rapid rotation at the stroke reversal that may lead to increased aerodynamic forces due to unsteady aerodynamic mechanisms (see Section 3). Furthermore, static bending tests by Combes and Daniel [10,11] showed anisotropy of wing structures in a variety of insect species. More recently, Mountcastle and Daniel [226] investigated the influence of wing compliance on the mean advective flows (indicative of induced flow velocity) using PIV techniques. Their results demonstrated that flexible wings yield mean advective flows with substantially greater magnitudes and orientations more beneficial to lift than those of stiff wings.

Fig. 18. (Continued)
While Section 4 reviewed studies in rigid flapping wing aerodynamics, this section presents a review of some recent efforts in flexible flapping wing aerodynamics/aeroelasticity focusing on chordwise-only flexible, spanwise-only flexible, and combined chordwise/spanwise flexible structures in that order. As summary, the recent studies are listed in Tables A1–A3 with key information.

5.1. Chordwise-flexible wing structures

Zhu [227] performed a fully coupled fluid–structure interaction analyses to investigate a chordwise flexible airfoil prescribed with pure plunge motion. To clarify the role of inertia on the deformation, the wing was studied in both water and air. Results showed that when the wing was immersed in air, the chordwise flexibility reduces both the thrust and the propulsion efficiency. However, when the wing was immersed in water, it increased the efficiency. Refs. [32,228–231] presented studies on both rigid and partially chordwise flexible airfoils prescribed with both pure plunge and/or combined plunge/pitch motions in water and showed that flexible wings may be more efficient than the rigid ones. In particular, Pederzani and Haj-Hariri [232] suggested that lighter plunging airfoils were capable of generating more thrust than heavier ones and were more efficient. They performed computational analyses on a rigid wing from which a portion was cut out and covered with a very thin and flexible material (latex) and showed that due to a snapping motion (i.e. non zero velocity in the direction opposite to that of the following stroke) of the latex at the beginning of each stroke, the strength of the vortices that were shed was higher in lighter wing structures, leading to the generation of more thrust. Furthermore, such structures required less input power in order to be snapped than heavier ones. Chaithanya and Venkatraman [233,234] investigated the influence of inertial effects due to prescribed motion on the thrust coefficient and propulsive efficiency of a plunging/pitching thin plate using inviscid flow theory and beam equations. Their results demonstrated that flexible airfoils with inertial effects yielded more thrust than those without inertial effects. This was due to the increase in the fluid loading in the former which subsequently led to an increase in the deformation. Due to their shape, deformed airfoils produced a force component along the forward velocity direction [234].

Gopalakrishnan [235] analyzed the effects of elastic cambering of a rectangular membrane flapping wing on aerodynamics in the forward flight using a linear elastic membrane solver coupled with an unsteady LES method. Different membrane pre-stresses were investigated to give a desired camber in response to the aerodynamic loading. The results showed that the camber introduced by the wing flexibility increased the thrust and lift production considerably. Analysis of flow structures revealed that the LEV stayed attached on the top surface of the wing, followed the camber, and covered a major part of the wing, which resulted in high force production. On the other hand, for rigid wings (which were also considered) the leading edge vortex lifted off from the surface resulting in low force production. To evaluate the role played by the LEV for a flexibly cambered airfoil, Gulcat [236] investigated (1) a thin rigid plate in a plunging motion; (2) a flexibly cambered airfoil whose camber was changed periodically; and (3) the plunging motion of a flexibly cambered airfoil. The leading edge suction force for all cases was predicted by means of the Blasius theorem and the time-dependent surface velocity distribution of the airfoil is determined by unsteady aerodynamic considerations. Gulcat [236] reported that the viscous effects obtained by the unsteady boundary-layer solution show very little alteration to the oscillatory behavior of the net propulsive force. They only reduced the amplitude of the leading edge suction force obtained by the unsteady aerodynamic theory. It was found that the major contribution to the thrust is due to heaving plunging; therefore, it is possible to get high propulsion efficiency with limited camber flexibility.
showed that thrust-indicative wake structures were observed behind the trailing edge of the airfoil for airfoils with flexure amplitudes of 0.0–0.5 of the chord length. It was shown that this wake structure evolved into a drag-indicative form as the flexure amplitude of the airfoil was increased to 0.6 and 0.7 of the chord length. Studies conducted under various combinations of Reynolds number and reduced frequency showed that the propulsive efficiency of a chordwise flexible airfoil in pure plunge was influenced primarily by the value of the reduced frequency rather than by the Reynolds number. Toomey and Eldredge [238] performed numerical and experimental investigations to understand the role of flexibility in flapping wing flight using two rigid elliptical sections connected by a hinge with torsion spring. The section at the leading edge was prescribed with fruit fly-like hovering wing kinematics [50], while the trailing edge section responded passively due to the fluid dynamic and inertial/elastic forces. It was found that the lift force and wing deflection are primarily controlled by the nature of the wing rotation. Faster wing rotation, for example, led to larger peak deflection and lift generation. Advanced rotation also led to a shift in the instant of peak wing deflection which increased the mean lift. In contrast to the rotational kinematics, the translational kinematics were shown to have very little impact on spring deflection or force. And, while the former was shown to be nearly independent of Reynolds number, the latter was shown to increase with increasing Reynolds number. Poirel et al. [239] conducted a wind tunnel experimental investigation of self-sustained oscillations of an aeroelastic NACA0012 airfoil occurring in the transitional Re regime, in particular, aeroelastic limit cycle oscillations for the airfoil constrained to rotate in pure pitch. The structural stiffness as well as the position of the elastic axis were varied. Their investigation suggested that laminar separation plays a role in the oscillations, either in the form of trailing edge separation or due to the presence of a laminar separation bubble.

Miao and Ho [237] prescribed a time-dependent flexible deformation profile for an airfoil in pure plunge and investigated the effect of flexure amplitude on the unsteady aerodynamic characteristics for various combinations of Reynolds number and reduced frequency. For a specific combination of Reynolds number, reduced frequency, and plunge amplitude, the results showed that thrust-indicative wake structures were observed behind the trailing edge of the airfoil for airfoils with flexure amplitudes of 0.0–0.5 of the chord length. It was shown that this wake structure evolved into a drag-indicative form as the flexure amplitude of the airfoil was increased to 0.6 and 0.7 of the chord length. Studies conducted under various combinations of Reynolds number and reduced frequency showed that the propulsive efficiency of a chordwise flexible airfoil in pure plunge was influenced primarily by the value of the reduced frequency rather than by the Reynolds number. Toomey and Eldredge [238] performed numerical and experimental investigations to understand the role of flexibility in flapping wing flight using two rigid elliptical sections connected by a hinge with torsion spring. The section at the leading edge was prescribed with fruit fly-like hovering wing kinematics [50], while the trailing edge section responded passively due to the fluid dynamic and inertial/elastic forces. It was found that the lift force and wing deflection are primarily controlled by the nature of the wing rotation. Faster wing rotation, for example, led to larger peak deflection and lift generation. Advanced rotation also led to a shift in the instant of peak wing deflection which increased the mean lift. In contrast to the rotational kinematics, the translational kinematics were shown to have very little impact on spring deflection or force. And, while the former was shown to be nearly independent of Reynolds number, the latter was shown to increase with increasing Reynolds number. Poirel et al. [239] conducted a wind tunnel experimental investigation of self-sustained oscillations of an aeroelastic NACA0012 airfoil occurring in the transitional Re regime, in particular, aeroelastic limit cycle oscillations for the airfoil constrained to rotate in pure pitch. The structural stiffness as well as the position of the elastic axis were varied. Their investigation suggested that laminar separation plays a role in the oscillations, either in the form of trailing edge separation or due to the presence of a laminar separation bubble.

Vanella et al. [89] conducted numerical investigations on a similar structure and found that the best performance (up to approximately 30% increase in lift) was realized when the wing was excited by a non-linear resonance at 1/3rd of its natural frequency. For all Reynolds numbers considered, the wake capture mechanism was enhanced due to a stronger flow around the wing at stroke reversal, resulting from a stronger vortex at the trailing edge. Heathcote et al. [240] investigated the effect of chordwise flexibility on aerodynamic performance of an airfoil in pure plunge under hovering conditions. Because the trailing edge was a major source of shedding of vorticity at zero freestream velocity, they showed that the amplitude and phase angle of the motion of the trailing edge affected the strength and spacing of the vortices, and the time averaged velocity of the induced jet. Direct force measurements confirmed that at high plunge frequencies, the thrust coefficient of the airfoil with intermediate stiffness was highest, although the least stiff airfoil can generate larger thrust at low frequencies. It was suggested that there was an optimum airfoil stiffness for a given plunge frequency and amplitude. Similar conclusions were made in another study [241] wherein, the influence of resonance on the performance of a chordwise flexible airfoil prescribed with pure plunge motion at its leading edge was studied. It was shown that while the mean thrust could increase with an increase in flexibility, below a certain threshold the wing is too flexible to communicate momentum to the flow. On the other hand, too much flexibility led to a net drag and hence, only a suitable amount of flexibility was desirable for thrust generation. Du and Sun [242] numerically investigated the effect of prescribed time-varying twist and chordwise deformation on the aerodynamic force production of a fruitfly in hover. The results showed that aerodynamic forces on the flapping wing were not affected much by the twist, but by the camber deformation. The effect of combined camber and twist...
deformation was found to be similar to that of camber deformation alone. With a deformation of 6% camber and 20\(\frac{1}{2}\) twist (typical values observed for wings of many insects), the lift increased by 10–20% compared to the case of a rigid flat plate wing. It was therefore shown that chordwise deformation could increase the maximum lift coefficient of a fruit fly wing model and reduce its power requirement for flight.

While most of the recent computational and experimental studies explored the role of wing flexibility in augmenting aerodynamic performance while focusing on single wings at relatively higher Reynolds numbers, Miller and Peskin [243] numerically investigated the effect of wing flexibility on the forces produced during clap and fling/peel motion [244] of a small insect (\(Re_{E} \approx 10\)) focusing on wing–wing interactions. They prescribed both clap and fling kinematics separately to a rigid and a chordwise flexible wing and showed that while lift coefficients produced during the rigid and flexible clap strokes were comparable, the peak lift forces in the flexible cases were higher than in the corresponding rigid cases. This was due to the peel motion which delayed the formation of the trailing edge vortices, thereby maintaining vortical asymmetry and augmenting lift for longer periods [41].

Zhao et al. [245] investigated the chordwise flexibility effects using 16 different dynamically scaled mechanical models of a fruit fly wing. Based on the prescribed kinematics, the wing was started impulsively from rest and moved over a 180\(\frac{1}{2}\) arc in 1.5 s. Each of the 16 wing models (each with a different chordwise bending stiffness) was tested at 23 different static angles of attack at the leading edge from \(-9\) to \(90\)\(\frac{1}{2}\) in 4.5\(\frac{1}{2}\) increments. Results from the experiments showed that the overall aerodynamic performance (lift generation) of flapping wings deteriorated as they became more flexible. However, it was shown that flexible wings could generate more lift at higher static angles of attack than rigid wings due to bending of the wing surface causing a slight enhancement in the lift to drag ratios for such wings over their rigid counterparts.

In another study, Heathcote and Gursul [32] performed water tunnel studies (see Fig. 25) to examine the thrust and efficiency of a flexible 2-D airfoil plunging in forward flight. Subsequently, computations were done by Aono et al. [246] using a nonlinear planar beam solver [247,248] and a NS solver [122] under the assumption of laminar flow. The airfoil consisted of a rigid teardrop element made out of aluminum and a thin flexible flat plate made out of steel (see Fig. 25). Experiments were conducted on several wing models by considering variations in the plate thickness. The key parameters related to all the cases considered in the experiment are included in Table 3. The computations were done on some of the experimental wing models and for a specific Reynolds number. The flexible flat plate was modeled as a cantilevered beam fixed at the trailing edge of the teardrop. In all the cases considered in both experiments and computations, the leading edge of the teardrop was actuated by a sinusoidal plunge displacement profile. Three different variations are considered for the plate thickness in the computations: 3.81 \times 10^{-3} m ("Rigid"), 1.27 \times 10^{-4} m ("Flexible"), and 5.04 \times 10^{-5} m ("Very Flexible"). The results presented here will
focus on computations while still including the corresponding experimental data.

Fig. 26 (A) shows the thrust coefficient as a function of normalized time (with respect to the period of plunge) for the “Rigid”, “Flexible”, and “Very Flexible” cases. As seen in the figure, the amplitude of instantaneous thrust increases with increasing flexibility. Fig. 26(B) shows the time history of total aerodynamic force coefficient (total force including both lift and thrust) for all three cases. Unlike in the case of the thrust coefficient, the instantaneous amplitude of the resultant aerodynamic force produced by the “Rigid” and “Flexible” wings is very close to each other at all time instants, while being very different from that of the “Very Flexible” wing. This behavior is akin to that of the lift coefficient (not included in the figure) whose amplitude is the largest in the “Rigid” case and smallest in the “Very Flexible” one. This is due to the redistribution of the total aerodynamic force between the lift and the thrust directions. Fig. 27 shows the mean thrust coefficient plotted as a function of the effective stiffness ($P_1$) for all variations of chordwise flexibility including both experimental and computational data. In the figure, each data point corresponds to a particular value of plate thickness (which in turn corresponds to a particular $P_1$). In general, within the range of flexibility considered in the computations, the mean thrust coefficient increases with increasing flexibility. However, for the cases considered in the experiments wherein more variations in flexibility were considered, the mean thrust initially increases with increasing flexibility but decreases in the most flexible case. Fig. 29 shows the time history of the effective angle of attack (illustrated in Fig. 28) for the three variations of flexibility considered in the computations. Fig. 29(A) corresponds to the effective angle of attack considering the whole airfoil (i.e. with reference to the leading edge point of the teardrop) and Fig. 29(B) to the angle considering only the flexible flat plate (i.e. with reference to the trailing edge point of the teardrop). And, Fig. 29(C) shows the instantaneous airfoil shape for all three flexible wings at selected time instants as illustrated under each snapshot. As seen in the figures, in general, with increasing chordwise flexibility, the effective angle of attack decreases. Considering the time instant corresponding to the middle of the downstroke (i.e. at $t/T=0.25$), it is seen in the figure that the effective angle of attack in the “Rigid” and “Flexible” cases is more than that of the “Very Flexible” case. This is because, with increasing chordwise deformation (as seen in Fig. 29(C), it is the largest in the “Very Flexible” case), the angle “$\alpha$” increases which subsequently results in a decrease of the effective angle of attack “$\alpha_e$”.

In order to estimate the individual contribution of the teardrop and the flexible plate to force generation, the time histories of thrust coefficient are shown in Fig. 30 separately for each case. It is seen in the figure that the thrust response with variation in flexibility is different in each of the two cases: with increasing
chordwise flexibility of the plate, the instantaneous thrust acting on the teardrop reduces because of a decrease in the leading edge suction peak (Fig. 31(C)). However, with increasing flexibility from “Rigid” to the “Very Flexible” case, the instantaneous thrust contributed by the flexible plate increases. This is because, the chordwise deformation of the rear flexible plate in both “Flexible” and “Very Flexible” cases result in an effective projected area for the thrust forces to develop. Streamlines and pressure contours around the three wing configurations are shown in Fig. 31 at the time instant $t/T=0.25$ that corresponds to the middle of down stroke. Furthermore, in Fig. 31 (A–2), (B–2), and (C–2), the pressure coefficient on the airfoil is plotted as a function of the chordwise position. The grey vertical line in the plots represents the boundary between the tear drop and the flexible thin plate. As seen in these figures, the leading edge suction is more dominant in the “Rigid” and “Flexible” cases than in the “Very Flexible” case.

Fig. 23. Comparison of near-field flow fields between a hawkmoth, a honeybee, a fruit fly, and a thrips around the middle of the downstroke. Wing-body computational models of (A) a hawkmoth model, (B) a honeybee model, (C) a fruit fly model, and (D) a thrips model, with the LEVs visualized by instantaneous streamlines and the corresponding velocity vectors in a plane cutting through the left wing at 60% of the semi-span.
Considering the outcome from the studies conducted on chordwise flexible plunging/pitching structures discussed so far, three factors may be identified that play a vital role in aerodynamic force generation in hover/forward flight: (i) airfoil plunge motion modifies the effective angle of attack and aerodynamic forces; (ii) relative moment of leading edge and trailing edge creates the pitch angle which dictates the direction of the net force; and (iii) airfoil shape deformation modifies effective geometry such as camber.

5.2. Spanwise-flexible wing structures

Numerical simulations were performed by Liu and Bose [248] for a 3-D pitching and plunging wing in forward flight. Their results showed that the phase of the flexing motion of the wing relative to the prescribed heave motion plays a key role in determining thrust and efficiency characteristics of the fin. Zhu [227] performed numerical investigations on a thin flapping foil prescribed with pure plunge motion in forward flight in different both air and water. His results showed that when the wing was immersed in air, the spanwise flexibility (through equivalent plunge and pitch flexibility) increased the thrust without an efficiency reduction and when the same wing was immersed in water, the spanwise flexibility reduced both the thrust and efficiency.

Heathcote et al. [250] conducted water-tunnel studies to study the effect of spanwise flexibility on the thrust, lift, and propulsive efficiency of a plunging flexible wing configuration in forward flight. A schematic of the experimental setup is shown in Fig. 25. Three wings of 0.3 m span and 0.1 m chord with varying levels of flexibility were constructed ("Inflexible", "Flexible", and "Highly Flexible", see Fig. 25(C)). The leading edge at the wing root was actuated with a prescribed sinusoidal plunge displacement profile. Wing shape was recorded with a 50-frames-per-second, high-shutter-speed, digital video camera. Overall wing thrust coefficient and tip displacement response were measured. The representations of the cross-section constructions are reproduced in Fig. 25(C). While the "Inflexible" wing cross-section was built-up from nylon and reinforced by steel rods, the "Flexible" and "Highly Flexible" cross-sections were built-up from PDMS (rubber) and stiffened with a thin metallic sheet made out of steel and aluminum respectively. Table 4 shows the key parameters related to the test cases. In a subsequent effort, computations were conducted on these wing configurations by Chimakurthi et al. [251] and Aono et al. [252] only at the chord-based Reynolds number of $3 \times 10^4$ using an aeroelastic framework developed by coupling a 3-D NS solver [122] with a nonlinear beam solver [253]. The flow is assumed to be laminar. It might be noted that in the computations, the PDMS around the metallic stiffeners was neglected. While the contribution of it to the overall stiffness of the wings will be negligible (due to a relatively very low Young’s modulus of the material compared to either steel or aluminum), the effect of it on the mass properties might be significant. Furthermore, it might also be noted that, the bending stiffness of the Highly Flexible wing was adjusted due to some discrepancies found by the experimentalists who conducted static loading tests and revealed the actual stiffness data via private communication.

Fig. 32 shows the time history of the instantaneous thrust coefficient and that of the total aerodynamic force coefficient for...
all three wing configurations for the case of $Re=3.0 \times 10^4$. In the figure, the experimental data is indicated as “EXP” and the computational data as “COMP”. From Fig. 33(A), it is seen that the “Flexible” wing case generates the highest thrust peaks, while in the “Very Flexible” wing case, the smallest. Similarly, as seen in Fig. 32(B), the amplitude of the total aerodynamic force coefficient is the largest in the “Flexible” wing and the smallest in the “Very Flexible” one. Furthermore, there is also a difference in phase among the three wing cases. Fig. 33 shows the mean thrust coefficient for all three wings plotted as a function of the non-dimensional parameter “effective stiffness (dimensionless parameter $P_1$)”. As shown in the figure, the mean thrust response is non-monotonic. In particular, it is observed that the largest and smallest mean thrust coefficients are produced by the “Flexible” and the “Very Flexible” wing cases, respectively. It might be noted that in all these cases, while there is an overall agreement between the experiments and the computations, there are noticeable discrepancies in the “Very Flexible” case due to
uncertainties in both modeling (mass density of PDMS was not accounted for in the computations) and experiments (material properties). Interested readers might refer to Chimakurthi et al. [251] and Aono et al. [252] for additional details about computational results and their correlations with experimental data.

The reasons for the diminishment of the lift/thrust forces in the “Very Flexible” wing case and the enhancement of the same in the “Flexible” one are as follows. Fig. 34 shows the plot of the instantaneous angle of attack along the span position at the time instant \( t/T = 0.25 \) (this is the time instant corresponding to the middle of downstroke during which the maximum lift/thrust forces are produced in the “Rigid” and “Flexible” cases) for all three plunging wing configurations. As seen from the plot, the area under the “Flexible” wing curve is the largest and the one under the “Very Flexible” one, the smallest. Since it is the cumulative effect of the angle of attack seen by the flow at all sections through the span that is important in total force generation on the wing, the forces in the “Flexible” case are the largest and those on the “Very Flexible” wing, the smallest. The decrease in the angle of attack in the case of the “Very Flexible” wing (at several sections through the wing) over that of the others led to a decrease in the leading edge suction which in turn caused a loss of total lift/thrust on the wing.

Sample pressure contours for all four wing configurations at the mid-span section at time instant \( t/T = 0.25 \) are shown in Fig. 35. The dominance of leading edge suction in the “Flexible” case and the reduction of it in the “Very Flexible” case are visible in that figure. The leading edge suction is obtained due to the normal projected area contributed by the leading edge curvature of the wing (see Fig. 35) that supports the generation of the horizontal force. The phase lag between the prescribed motion

---

**Fig. 26.** Time response of thrust \((C_T)\) and total aerodynamic force coefficients \((C_f)\) of the chordwise flexible wing structures at \( Re = 9.0 \times 10^3 \) and \( St = 0.34 \): (A) Thrust coefficient; (B) Total aerodynamic force coefficient.
and the deformation of the wing is another quantity that could be used to explain the aerodynamic force generation in flexible flapping wings [251,252]. The phase lag at the wing tip with respect to the prescribed motion for the “Rigid”, “Flexible”, and “Very Flexible” cases are, 0, 25.7°, and 126.3°, where the in-phase percentages are 100%, 86%, and 24%, respectively. As seen in the table, the “Very Flexible” wing has the largest phase lag and the shortest time of in-phase motion at the wing tip in a stroke. In contrast, the “Flexible” wing has the smallest phase lag and the longest in-phase motion in a stroke. As a result of the substantial phase lag in the “Very Flexible” case, the wing tip and root move in opposite directions during most of the stroke resulting in lower effective angles of attack and consequently lesser aerodynamic force generation. This is seen by correlating the phase lag information with the mean thrust response of the “Flexible” and the “Very Flexible” wing cases in Fig. 32(A).

The results presented so far focused on a single case of the Reynolds number $3 \times 10^4$. Results related to parametric studies are available in Heathcote et al. [250]. To summarize the same, it was found that the mean thrust coefficient is a function of the Strouhal number (based on the amplitude of the mid-span of the wing), and is very weakly dependent on the Reynolds number $Re$.  

5.3. Combined chordwise-and-spanwise flexible wing structures

Watman and Furukawa [254] investigated the effects of passive pitching motions of flapping wings on the aerodynamic performance using robotic wing models. They considered two types of passive flapping wing models: first model used a rigid connection between all parts of the structure. This design was utilized in several MAVs [1,5,6] and served as a common design used for analysis of flapping wings. Second model was designed in a way to allow free rotation of ribs and membrane over a limited angle. This design was used recently in a small MAV prototype [255]. They showed that the former passive flapping wing (constrained) was found to have better performances compared to latter one due to favorable variation of passive pitching angle of the wing. Hui et al. [256] examined various flexible wing structures to evaluate their implications on flapping wing aerodynamics. They showed that the flexible membrane wings were found to have better overall aerodynamic performance (i.e., lift-to-drag ratio) over the rigid wing for soaring flight, especially for high-speed soaring flight or at relatively high angle of attack. The rigid wing was found to have better lift production performance in general. The latex wing, which is the most flexible among the three tested wings, was found to have the best thrust generation performance for flapping flight. The less flexible nylon wing, which has the best overall aerodynamic performance for soaring flight, was found to be the worst for flapping flight applications. Shkarayev et al. [257] investigated the aerodynamics of cambered membrane flapping wings. Specifically, a cambered airfoil was introduced into the wing by shaping metal ribs attached to the membrane skin of the 25 cm-wing-span model. The thrust force generated by a 9% camber wing was found to be 30% higher than that of the same size flat wing. Adding a dihedral angle to the wings and keeping
the flapping amplitude constant improved the cambered wing's performance even further. The aerodynamic coefficients are defined using a reference velocity as a sum of two components: a free stream velocity and a stroke-averaged wing tip flapping velocity. The lift, drag, and pitching moment coefficients obtained using this procedure collapse well for studied advance ratios, especially at lower angles of attack. Kim et al. [258] developed a biomimetic flexible flapping wing using micro-fiber composite actuators and experimentally investigated the aerodynamic performance of the wing under flapping and non-flapping motion in a wind tunnel. Results showed that the camber due to wing flexibility could produce positive effects (i.e. stall delay, drag reduction, and stabilization of the LEV) on flapping wing aerodynamics in quasi-steady and unsteady region. Mueller et al. [259] presented a new versatile experimental test for measuring the thrust and lift of a flapping wing MAV. They showed increase in average thrust due to increased wing compliance and detrimental influence of excessive compliance on drag forces during high-frequency operation. Also they observed the useful effect of compliance on the generation of extra thrust at the beginning and end of flapping motions. Hamamoto et al. [260] conducted a fluid–structure interaction analysis on a deformable dragonfly wing in hover and examined the advantages and disadvantages of flexibility. They tested three types of flapping flight: a flexible wing driven by dragonfly flapping motion, a rigid wing (stiffened version of the original flexible dragonfly wing) driven by dragonfly flapping motion, and a rigid wing driven by modified flapping based on tip motion of the flexible wing. They found that the flexible wing with nearly the same average energy consumption generated almost the same amount of lift force as the rigid wing with modified flapping motion, which realized the same angle of attack at the aerodynamically dominant sections of the wing. However, the rigid wing required 19% more peak torque and 34% more peak power, indicating the usefulness of wing flexibility. Luo et al. [261] have simulated the interaction between a viscous, unsteady flow and deformable thin structures to simulate a dragonfly wing.
They focused on the situations where the wing inertia is large compared to the fluid forces, and therefore, the flow–structure interaction is essentially unidirectional (from the structure to the flow, and not the other way around.) However, the two-way interaction is built in the code, and it only takes longer time to perform the simulations. Aono et al. [262] reported a combined computational and experimental study of a well-characterized flapping wing structure was conducted. An aluminum wing was prescribed with single degree-of-freedom flapping at 10 Hz frequency and $\pm 21^\circ$ amplitude. Flow velocities and deformation were measured using digital image correlation and digital PIV techniques, respectively. In the most flexible flapping wing case, the elastic twisting of the wing was shown to produce substantially larger mean and instantaneous thrust due to shape deformation-induced changes in effective angle of attack. Relevant fluid physics were documented including the counter-rotating vortices at the leading and the trailing edge, which interacted with the tip vortex during the wing motion.

Singh and Chopra [263] developed an experimental apparatus with a bio-inspired flapping mechanism to measure the thrust generated for a number of wing designs. The key conclusions that stemmed from this study were that the inertial loads constituted the major portion of the total loads acting on the flapping wings tested on the mechanism and that for all the wings tested, the

Table 4
Flow, geometrical, and structural properties along with dimensionless parameters associated to the spanwise flexible wing cases in Refs. [240–242].

<table>
<thead>
<tr>
<th></th>
<th>Rigid</th>
<th>Flexible</th>
<th>Very flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference velocity [m/s]</td>
<td>$U_{ref}=U_{\infty}$</td>
<td>$3.0 \times 10^{-1}$</td>
<td>$3.0 \times 10^{-1}$</td>
</tr>
<tr>
<td>Water density [kg/m$^3$]</td>
<td>$\rho_w$</td>
<td>$1.0 \times 10^4$</td>
<td>$1.0 \times 10^3$</td>
</tr>
<tr>
<td>Mean chord length [m]</td>
<td>$c_m$</td>
<td>$1.0 \times 10^{-1}$</td>
<td>$1.0 \times 10^{-1}$</td>
</tr>
<tr>
<td>Semi-span [m]: $R$</td>
<td>$h$</td>
<td>$3.0 \times 10^{-3}$</td>
<td>$3.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Thickness of wing [m]</td>
<td>$h_c$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Young's modulus [Pa]: $E$</td>
<td>$E$</td>
<td>$2.1 \times 10^{11}$</td>
<td>$4.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Density of structure [kg/m$^3$]: $\rho_s$</td>
<td>$\rho_s$</td>
<td>$7.8 \times 10^{3}$</td>
<td>$2.7 \times 10^{3}$</td>
</tr>
<tr>
<td>Plunge amplitude [m]: $h_a$</td>
<td>$h_a$</td>
<td>$1.75 \times 10^{-2}$</td>
<td>$1.75 \times 10^{-2}$</td>
</tr>
<tr>
<td>Plunge frequency [Hz]: $f$</td>
<td>$f$</td>
<td>$1.74 \times 10^5$</td>
<td>$1.74 \times 10^5$</td>
</tr>
<tr>
<td>$Re$</td>
<td>$3.0 \times 10^4$</td>
<td>$3.0 \times 10^4$</td>
<td>$3.0 \times 10^4$</td>
</tr>
<tr>
<td>$St$</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1$</td>
<td>2.14 $\times 10^2$</td>
<td>4.07 $\times 10^1$</td>
</tr>
<tr>
<td>$I_{1}$</td>
<td>Infinite</td>
<td>7.8</td>
<td>2.7</td>
</tr>
</tbody>
</table>
thrust dropped at higher frequencies. Further, the author found that at such frequencies, the light-weight and highly flexible wings used in the study exhibited significant aeroelastic effects. Young et al. [121] conducted numerical investigations on a tethered desert locust called *Schistocerca gregaria* [118]. Results demonstrated that time-varying wing twist and camber were essential for the maintenance of attached flow. The authors emphasized that while high-lift aerodynamics was typically associated with massive flow separation and large LEVs, when high lift was not required, attached flow aerodynamics could offer greater efficiency. Their results further showed that, in designing robust lightweight wings that could support efficient attached flow, it was important to build a wing that undergoes appropriate aeroelastic wing deformation through the course of a wing beat.

Agrawal and Agrawal [264] investigated the benefits of insect wing flexibility on flapping wing aerodynamics based on experiments and numerical simulations. They compared the performance of two synthetic wings: (i) a flexible wing based on a bio-inspired design of the hawkmoth (*Manduca Sexta*) wing and (ii) a rigid wing of similar geometry. The results demonstrated that more thrust was generated by the bio-inspired flexible wing compared to the rigid wing in all wing kinematic patterns considered. They emphasized that the results provided motivation for exploring the advantages of passive deformation through wing flexibility and that coupled fluid–structure simulations of flexible flapping wings were required to gain a fundamental understanding of the physics and to guide optimal flapping wing MAV designs.

As a flight vehicle exploring the merit of aeroelasticity effect, Jones and Platzer [265] and Jones et al. [266] devised a flapping concept which allowed the wing to flap with a constant amplitude to produce thrust from root to tip. In a follow-up study conducted by Kaya et al. [267], the flapping motion was optimized for maximum thrust. Specifically, the flapping airfoils were attached to swing arms by an elastic joint, which could be treated as a spring-mass system. The stiffness coefficient and the mass

**Fig. 32.** Effect of spanwise flexibility on instantaneous aerodynamic force generation at $Re=3.0 \times 10^4$ and $St=0.2$: (A) Thrust ($C_T$). (B) Total aerodynamic force ($C_f$).
moment of inertia of the airfoil were then optimized using the gradient-based method for maximum thrust. The optimization results showed that the aeroelastic pitching with optimum elastic properties produces significantly higher thrust than that of an optimum sinusoidal pitching. The aeroelastic pitching provided lower effective incidence angles and delayed the leading edge vortex formation, which in turn augmented the thrust generation. Wu et al. [268] and Sällström et al. [269] presented a multi-disciplinary experimental endeavor correlating flapping wing MAVs aeroelasticity and thrust production by quantifying and comparing elasticity, dynamic responses, and air flow patterns of six different pairs of MAV wings (in each one, the membrane skin was reinforced with different leading edge and batten configurations) of the Zimmerman planform (two ellipses meeting at the quarter chord) with varying elastic properties. In their experiment, single degree-of-freedom flapping motion was prescribed to the wings in both air and vacuum. Amongst many conclusions, they found that, within the range of flexibility considered, more flexible wings are more thrust-effective at lower frequencies whereas stiffer wings are more effective at higher frequencies. They hypothesized that flexible wings may have a certain actuation frequency for peak thrust production and the performance would degrade once that frequency is passed.

These studies as well as the investigations of Heathcote et al. [32,250], Chimakurthi et al. [251], Aono et al. [252], and Tang et al. [230] offer consistent findings. It seems like the spanwise flexibility increases aerodynamic forces by creating higher effective angles of attack via spanwise deformation. However, apart from affecting the overall aerodynamic force generation, the chordwise flexibility can redistribute lift versus thrust by changing the projection angle of the wing with respect to the freestream by changing airfoil via camber deformation, for example. Of course, similar behavior may be observed in spanwise flexible with passive twist deformation along the wing span. Overall, both spanwise and chordwise flexibility need to be considered together in order to optimize the aerodynamic performance under different flight speed and environment uncertainty such as wind gust.

6. Conclusions

In this article, a number of previous efforts focusing on flapping wing aerodynamics and aeroelasticity have been highlighted. Furthermore, important features related to the aerodynamics associated with rigid and flexible flapping wings have been reviewed.
For $Re=O(10^5)$, corresponding to the small insect flight regime, the aerodynamic performances associated with various wing geometry and flapping kinematics have been with extensive comparison between 2-D and 3-D studies, it has been observed that:

(i) There is significant variance in the spanwise distribution of forces in the 3-D cases. Cases which suppress the TiV generation, those with the highest pitching amplitudes and thus low angles of attack, and small translational velocity when vertical (e.g. near synchronized rotation), appear to have a relatively constant response along the span. In contrast, 3-D cases with a prominent TiV exhibit significant variations along the span which do not have a strong correlation to the 2-D lift values experienced.

(ii) At Reynolds number at 65, the 3-D fluid physics and TiV effects are able to augment the lift by: (a) the presence of a low pressure region near the tip and, (b) the anchoring of an otherwise shed vortex (in 2-D) near the tip.

(iii) The surrogate modeling techniques reveals that the hierarchy of variable sensitivity in the mean lift changes between 2-D and 3-D in space. In 2-D the importance is pitching angular amplitude, phase lag and normalized stroke amplitude. In 3-D the hierarchy switches to phase lag, pitching angular amplitude, and normalized stroke amplitude. Regions for which 2-D kinematics outperformed 3-D and vice versa are identified in the design space considered.

(iv) That a 3-D low aspect ratio wing can produce higher lift than a 2-D airfoil is not supported by the classical stationary wing theory [78], which suggests that TiVs decrease performance. In unsteady flow around a hovering wing, however, the TiVs can contribute to lift generation rather than just drag generation on the wing.

The LEV is common and important to the flapping wing aerodynamics at the Reynolds number of $O(10^5)$ or lower, which corresponds to the hummingbird and insect flight regimes. However, the LEV structures and distribution of spanwise flow inside the LEV change with the variation of Reynolds number (wing sizing, flapping frequency, etc.) and with the interactions between LEVs and TiVs and hence influence the aerodynamic force generation. In the hawkmoth and honeybee studies, at higher Reynolds numbers ($O(10^5)$ and $O(10^6)$), it is seen that the LEV has a spiral structure and breaks down at the middle of the downstroke. In the fruit fly studies, at lower Reynolds number ($O(10^3)$), it is observed that the LEV has an ordered structure and it connects with the TiV. A dependency on the Reynolds number is observed in both the spanwise flow inside the LEV and the spanwise variation of the pressure-gradient on the wing surfaces.

The LEV as a lift enhancement mechanism for flapping wing at Reynolds number of $O(10^5–10^6)$ seem less certain because, as pointed out in Ref. [222], a dynamic-stall vortex on an oscillating airfoil is often found to break away and to convect elsewhere as soon as the airfoil translates.

The impact of structural flexibility on aerodynamics undergoing plunging motion in forward flight is also highlighted as follows:

(i) The thrust generation consists of contributions due to both leading edge suction and the pressure projection of the chordwise deformed rear foil. When the rear foil's flexibility increases, the thrust of the teardrop element decreases, as well as the effective angle of attack.

(ii) Within a certain range as chordwise flexibility increases, even though the effective angle of attack and the net aerodynamic force are reduced due to chordwise shape deformation, both mean and instantaneous thrust are enhanced due to the increase of projected area normal to the flight trajectory.

(iii) For the spanwise flexible case, correlations of the motion from the root to the tip play a role. Within a suitably selected range of spanwise flexibility, the effective angle of attack and thrust forces of a plunging wing are enhanced due to the wing deformations.

While airfoil shape and spanwise geometry obviously affect the aerodynamics of a flapping wing, it seems from the growing body of the studies that the implications of structural flexibility are consistent with respect to different wing geometries.

The various scaling parameters vary with the length and time scales in different proportionality, the scaling invariance of both fluid dynamics and structural dynamics as the size changes is fundamentally difficult and challenging. It also seems that there is a desirable level of structural flexibility to support desirable aerodynamics. Significant work needs to be done to better understand the interaction between structural flexibility and aerodynamic performance under unpredictable wind gust conditions.

In summary, there have been many efforts ongoing to increase our understanding of the fluid physics of biology-inspired mechanisms that simultaneously provide lift and thrust, enable hover, and grant high flight control authority, while minimizing power consumption. Detailed experimental efforts and first principles-based computational modeling and analysis capabilities are essential in support of the investigation of issues related to fluid–structure interactions, unsteady freestream (wind gust), and unsteady aerodynamics. Other issues for further research are:

(i) In order to support the broad flight characteristics such as take-off, forward flight of varying speed, wind gust response, hover, perch, threat avoidance, station tracking, and payload variations, a variety of wing kinematics and body/leg maneuvers will need to be employed. Considering the variations in size and flapping patterns of natural flyers, there is significant potential and need for us to further refine our knowledge regarding the interplay between kinematics and aerodynamics.

(ii) It is established that local flexibility can significantly affect aerodynamics in both fixed and flapping wings. Furthermore, as already discussed above, insects' wing properties are anisotropic, with the spanwise bending stiffness about 1–2 orders of magnitude larger than the chordwise bending one. As the vehicle size changes, the scaling parameters cannot all maintain invariance due to different scaling trends associated with them. This means that if, for measurement precision, instrumentation preference, etc., one cannot do laboratory test of a flapping wing design using different sizes or flapping frequencies. A closely coordinated computational and experimental framework is needed in order to facilitate the exploration of the vast design space (which can have $O(10^5)$ or more design variables including geometry, material properties, kinematics, flight conditions, and environmental parameters) while searching for optimal and robust designs.

(iii) The implications of the dynamics and stability of a flapping vehicle [270–272], in association with flapping wing aerodynamics is still inadequately understood. In particular, the vehicle stability via passive shape deformation due to flexible structures needs to be addressed.

(iv) Bio-inspired mechanisms [273,274] need to be developed for the flapping wing. These mechanisms include, for example, joints and distributed actuation to enable flapping and morphing. Most importantly, these mechanisms should be capable of mitigating wind gust.
Studies of chordwise-flexible wing structures.

Table A1

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref no.</th>
<th>Model</th>
<th>Kinematics</th>
<th>Re</th>
<th>St or k</th>
<th>Structural information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heathcote and Gursul</td>
<td>[32]</td>
<td>Teardrop NACA0012</td>
<td>Plunge</td>
<td>9 × 10³</td>
<td>0.1–0.7</td>
<td>$E=205$ GPa, $\rho_f=7800$ kg/m³, $h_f=5 \times 10^{-3}$–3.8 × 10⁻⁴ m</td>
</tr>
<tr>
<td>Isihara, Horie and Denda</td>
<td>[88]</td>
<td>Shell/Beam Pitch/plunge</td>
<td>2.7 × 10⁵</td>
<td>0.054</td>
<td>–</td>
<td>Torsional stiffness: 0.8–154.4 cm²/(s²rad)</td>
</tr>
<tr>
<td>Vanella, Fitzgerald, Preidikman, Balaras</td>
<td>[89]</td>
<td>Beam Pitch/plunge</td>
<td>75</td>
<td>–</td>
<td>–</td>
<td>Frequency ratio: 1/2, 1/3, 1/4, and 1/6</td>
</tr>
<tr>
<td>Zhu</td>
<td>[227]</td>
<td>Plate Plunge</td>
<td>$2 \times 10^4$</td>
<td>0.2</td>
<td>–</td>
<td>$E=20–2000$ GPa, $\rho_f=7800$ kg/m³, $h_f=4 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Yamamoto, Terada, Nagamatu and Imazumi</td>
<td>[228]</td>
<td>Airfoils Pitch/plunge</td>
<td>$10^{6}$–$10^7$</td>
<td>–</td>
<td>–</td>
<td>Elasticity of flexible part: 3.3 mm/N–40 mm/N</td>
</tr>
<tr>
<td>Prempraneerach, Hover and Triantafyllou</td>
<td>[229]</td>
<td>NACA0014 Pitch/plunge</td>
<td>$4 \times 10^6$</td>
<td>0.1–0.45</td>
<td>–</td>
<td>Flexible rubber: $E=3.18–50.6$ GPa</td>
</tr>
<tr>
<td>Tang, Vieru and Shyy</td>
<td>[230]</td>
<td>Teardrop plate Plunge</td>
<td>$9.0 \times 10^5$</td>
<td>0.5</td>
<td>–</td>
<td>$E=205$ GPa, $\rho_f=7850$ kg/m³, $h_f=5 \times 10^{-2}$–3.8 × 10⁻⁴ m</td>
</tr>
<tr>
<td>Chandar and Damodaran</td>
<td>[231]</td>
<td>Teardrop plate Plunge</td>
<td>$9.0 \times 10^5$</td>
<td>0.5</td>
<td>–</td>
<td>$E=205$ GPa, $\rho_f=7850$ kg/m³, $h_f=5 \times 10^{-2}$–3.8 × 10⁻⁴ m</td>
</tr>
<tr>
<td>Pederzani and Haj-Hariri</td>
<td>[232]</td>
<td>Airfoils Plunge</td>
<td>$5.0 \times 10^2$</td>
<td>5.5</td>
<td>–</td>
<td>Membrane (latex): $\rho_f=0.5$–1.0 kg/m³, Fixed trailing edge or free trailing edge</td>
</tr>
<tr>
<td>Chaithanya and Venkatraman</td>
<td>[232,234]</td>
<td>Plate Pitch/plunge</td>
<td>Inviscid</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Gopalakrishnan</td>
<td>[235]</td>
<td>Rectangular membrane Pitch/plunge</td>
<td>$1.0 \times 10^4$</td>
<td>1.7</td>
<td>–</td>
<td>$E=114$ GPa, $\rho_f=4500$ kg/m³, $h_f=15 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>Gulcat and Ho</td>
<td>[236]</td>
<td>Thin airfoil NACA0012 Plunge</td>
<td>$10^2$–$10^5$</td>
<td>0.5–1.5</td>
<td>–</td>
<td>Prescribed camber</td>
</tr>
<tr>
<td>Toomey and Eldredge</td>
<td>[238]</td>
<td>Two ellipses Pitch/plunge</td>
<td>$1.2 \times 10^7$–6.1 × 10⁵</td>
<td>–</td>
<td>–</td>
<td>Prescribed wing deformation</td>
</tr>
<tr>
<td>Heathcote, Martin and Gursul</td>
<td>[32,240]</td>
<td>Teardrop Plunge</td>
<td>0.75–2.1 × 10⁴</td>
<td>0.17–0.40</td>
<td>–</td>
<td>$E=205$ GPa, $\rho_f=7800$ kg/m³, $h_f=5 \times 10^{-2}$–3.8 × 10⁻⁴ m</td>
</tr>
<tr>
<td>Michelin and Smith</td>
<td>[241]</td>
<td>Plate Plunge</td>
<td>Inviscid</td>
<td>0.08–0.8</td>
<td>–</td>
<td>Euler–Bernoulli equation: $P=10^{-2}$–$10^{-3}$ m</td>
</tr>
<tr>
<td>Du and Sun</td>
<td>[242]</td>
<td>3% flat plate fruit fly Pitch/ flap</td>
<td>$3.3 \times 10^2$ 6.0 × 10⁵</td>
<td>–</td>
<td>–</td>
<td>Prescribed wing deformation</td>
</tr>
<tr>
<td>Miller and Peskin</td>
<td>[243]</td>
<td>Beam Clap and fling</td>
<td>10</td>
<td>–</td>
<td>–</td>
<td>Bending stiffness: 6.9 × 10⁻⁷–1.1 × 10⁻⁷ N/m²</td>
</tr>
<tr>
<td>Zhao, Huang, Deng and Sane</td>
<td>[245]</td>
<td>Fruitfly-like Impulsive/pitch/ flap</td>
<td>$2.0 \times 10^5$</td>
<td>–</td>
<td>–</td>
<td>$EI_b=8.6 \times 10^{-6}$–5.3 × 10⁻⁴ m</td>
</tr>
</tbody>
</table>
Table A2

Studies of spanwise-flexible wing structures.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref. no.</th>
<th>Model</th>
<th>Kinematics</th>
<th>Re</th>
<th>St or k</th>
<th>Structural information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhu</td>
<td>[227]</td>
<td>Plate</td>
<td>Plunge</td>
<td>2.0 × 10⁴</td>
<td>0.2</td>
<td>E=20–2000 GPa, ρₓ=7800 kg/m³, hₓ=4 × 10⁻³ m</td>
</tr>
<tr>
<td>Liu and Bose, Heathcote, Wang</td>
<td>[249]</td>
<td>Immature fin like</td>
<td>Pitch/plunge</td>
<td>1–3.0 × 10⁴</td>
<td>0.05–0.9</td>
<td>(1) Inflexible, (2) Steel (E=210 GPa, ρₓ=7800 kg/m³, hₓ=1 × 10⁻³ m), (3) Aluminum (E=70 GPa, ρₓ=2700 kg/m³, hₓ=1 × 10⁻³ m)</td>
</tr>
<tr>
<td>Gursul</td>
<td>[250]</td>
<td>NACA0012</td>
<td>Plunge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chimakurthi, Tang, Palacios,</td>
<td>[251]</td>
<td>NACA0012</td>
<td>Plunge</td>
<td>3.0 × 10⁴</td>
<td>0.4–1.82</td>
<td>(1) Rigid, (2) Steel (E=210 GPa, ρₓ=7800 kg/m³, hₓ=1 × 10⁻³ m)</td>
</tr>
<tr>
<td>Cesnik and Shyy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aono, Chimakurthi, Cesnik, Liu</td>
<td>[252]</td>
<td>NACA0012</td>
<td>Plunge</td>
<td>3.0 × 10⁴</td>
<td>1.82</td>
<td>(1) Inflexible, (2) Flexible (E=210 GPa, ρₓ=7800 kg/m³, hₓ=1 × 10⁻³ m), (3) Very flexible (E=70 GPa, ρₓ=2700 kg/m³, hₓ=1 × 10⁻³ m)</td>
</tr>
<tr>
<td>and Shyy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A3

Studies of combined spanwise-and-chordwise flexible wing structures.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref. no.</th>
<th>Model</th>
<th>Re</th>
<th>St or k</th>
<th>Structural information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zheng, Wang, Khan, Vallance, Mittal and</td>
<td>[120]</td>
<td>Hawkmoth-like</td>
<td>5.0 × 10³</td>
<td>0.2–0.4</td>
<td>–</td>
</tr>
<tr>
<td>Hedrick</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young, Walker, Bompherey, Taylor and</td>
<td>[121]</td>
<td>Realist locust wings</td>
<td>4.0–5.0 × 10³</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Thomas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daniel and Combes</td>
<td>[232]</td>
<td>Beam</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Mountcastle and Daniel</td>
<td>[226]</td>
<td>Real moth wing</td>
<td>4.0–7.0 × 10³</td>
<td>0.2–0.3</td>
<td>–</td>
</tr>
<tr>
<td>Wu, Ifju, Stanford, Sällström, Ukeiley,</td>
<td>[268]</td>
<td>Zimmerman</td>
<td>1.0–3.0 × 10⁴</td>
<td>0.4</td>
<td>–</td>
</tr>
<tr>
<td>Love and Lind</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sällström, Ukeiley, Wu and Ifju</td>
<td>[269]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Watman and Furukawa</td>
<td>[252]</td>
<td>Half ellipse</td>
<td>5.0 × 10⁵</td>
<td>0.83</td>
<td>–</td>
</tr>
<tr>
<td>Hui, Kumar, Abate and Albertani</td>
<td>[256]</td>
<td>Bird-like</td>
<td>2.0–8.0 × 10⁴</td>
<td>0.6–3.3</td>
<td>–</td>
</tr>
<tr>
<td>Shkarayev, Silin, Abate and</td>
<td>[257]</td>
<td>Bird-like</td>
<td>1.6–4.8 × 10⁴</td>
<td>0.46–1.4</td>
<td>–</td>
</tr>
<tr>
<td>Albertani</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kim, Han and Kwon</td>
<td>[258]</td>
<td>Bird-like</td>
<td>2.0–3.0 × 10⁴</td>
<td>1–7</td>
<td>–</td>
</tr>
<tr>
<td>Muller, Bruck and Gupta</td>
<td>[259]</td>
<td>Bird-like</td>
<td>103–104</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Hamamoto, Ohtia, Hara, and Hisada</td>
<td>[260]</td>
<td>Dragonfly-like</td>
<td>103</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Luo, Yin, Dai and Doyle</td>
<td>[261]</td>
<td>Plate</td>
<td>50</td>
<td>0.94</td>
<td>–</td>
</tr>
<tr>
<td>Aono, Chimakurthi, Wu, Sällström, Stanford,</td>
<td>[262]</td>
<td>Zimmerman</td>
<td>2.6 × 10³</td>
<td>0.56</td>
<td>Reduced bending stiffness:5.63</td>
</tr>
<tr>
<td>Cesnik, Ifju, Ukeiley and Shyy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Singh and Chopra</td>
<td>[263]</td>
<td>Fruit fly-like mechanical</td>
<td>1.5 × 10⁴</td>
<td>0.31</td>
<td>Reduced bending stiffness:0.08</td>
</tr>
<tr>
<td>model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agrawal and Agrawal</td>
<td>[264]</td>
<td>Hawkmoth-like</td>
<td>7000</td>
<td>0.2</td>
<td>Carbon rods (E=200 GPa), nylon 6/6 rods (E=3 GPa), rubber (E=1.5 MPa), latex film (E=1.7 MPa), hₓ=0.1 × 10⁻³ m</td>
</tr>
</tbody>
</table>

References

Brachenbury J. Wing movements in the bush cricket.


Greenelewitz CH. Dimensional relationships for flying animals. Smithsonian Misc Collect 1962;136:1–146.


